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15 对亲和数

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摘要: 给出了 15 对亲和数^[9].

关键词: 数论; 亲和数; 完全数

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0 引言

寻找亲和数是数学中的一个有趣问题, 文献[5]中找到 90 亿以下的亲和数 1 360 对; 目前我们在 CAMPAQ 486 微机(主频 66)上用 TURBOC 2.0 编程语言又找到了超过 90 亿亲和数 15 对^[9].

定义 $\sigma(x)$ 是 x 的所有因子(含 1 和 x)之和, 如果 $\sigma(a) = \sigma(b) = a + b$, 其中 $a \neq b$, 则称 a 和 b 为亲和数.(如果 $a = b$, 则称 a 为完全数)^[13]例如:

$$\sigma(220) = \sigma(4 \times 5 \times 11) = \sigma(4) \times \sigma(5) \times \sigma(11) = 7 \times 6 \times 12 = 504$$

$\sigma(284) = \sigma(4 \times 71) = \sigma(4) \times \sigma(71) = 7 \times 72 = 504 = 220 + 284$. 220 与 284 是最小的亲和数对^[13]

最小的 12 对亲和数对是:

220 与 284:	1184 与 1210:	2620 与 2924:
5020 与 5564:	6232 与 6368:	1074 与 10856:
12285 与 14595:	17296 与 18416:	63020 与 76084:
66928 与 66992:	67095 与 71145:	69615 与 87633:

引理 1 如果 m 与 n 互素, 则 $\sigma(mn) = \sigma(m) \times \sigma(n)$.

引理 2 如果 p 为素数, 则 $\sigma(p^n) = 1 + p + \dots + p^n$. 特别地, 如果 $m = 2^n$ 则 $\sigma(m) = 2m - 1$

1 主要结果

我们通过 TURBOC 2.0 编程语言找到的 15 对大的亲和数是:

- 1) 111671419232 与 113704276768;
- 2) 111880731688 与 119211533912;
- 3) 112298547568 与 117329612432;
- 4) 112353092008 与 120518667992;

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- 5) 112362647384 与 120901152616; 6) 112617286336 与 114380684864;
 7) 113241732896 与 114941855968; 8) 113998485656 与 129475850344;
 9) 114172017608 与 124758957112; 10) 117242087552 与 118103736448;
 11) 120213482176 与 120876490304; 12) 124650153536 与 125392307008;
 13) 126131520448 与 128101671872; 14) 128564124736 与 129904568384;
 15) 128982259904 与 129143642944⁽¹³⁾

用上述引理验证以上 15 对亲和数如下:

- 1) $a = 32 \times 41 \times 1439 \times 59149$, $b = 32 \times 239 \times 389 \times 38219$,
 $\sigma(a) = 63 \times 42 \times 1440 \times 59150 = \sigma(b) = 63 \times 240 \times 390 \times 38220 = a + b^{(13)}$
 2) $a = 8 \times 11 \times 10609 \times 119839$, $b = 8 \times 31 \times 641 \times 749909$,
 $\sigma(a) = 15 \times 12 \times 10713 \times 119840 = \sigma(b) = 15 \times 32 \times 642 \times 749910 = a + b^{(13)}$
 3) $a = 16 \times 19 \times 383 \times 964499$, $b = 16 \times 127 \times 449 \times 128599$,
 $\sigma(a) = 31 \times 20 \times 384 \times 964500 = \sigma(b) = 31 \times 128 \times 450 \times 128600 = a + b^{(13)}$
 4) $a = 8 \times 19 \times 29 \times 109 \times 197 \times 1187$, $b = 8 \times 59 \times 131 \times 179 \times 10889$,
 $\sigma(a) = 15 \times 20 \times 30 \times 110 \times 198 \times 1188 = \sigma(b) = 15 \times 60 \times 132 \times 180 \times 10890 = a + b^{(13)}$
 5) $a = 8 \times 89 \times 109 \times 659 \times 2197$, $b = 8 \times 43 \times 269 \times 769 \times 1699^{(13)}$
 $\sigma(a) = 15 \times 90 \times 110 \times 660 \times 2380 = \sigma(b) = 15 \times 44 \times 270 \times 770 \times 1700 = a + b^{(13)}$
 6) $a = 64 \times 103 \times 167 \times 102299$, $b = 64 \times 10079 \times 177319^{(13)}$
 $\sigma(a) = 127 \times 104 \times 168 \times 102300 = \sigma(b) = 127 \times 10080 \times 177320 = a + b^{(13)}$
 7) $a = 32 \times 43 \times 6143 \times 13397$, $b = 32 \times 127 \times 2551 \times 11087$,
 $\sigma(a) = 63 \times 44 \times 6144 \times 13398 = \sigma(b) = 63 \times 128 \times 2552 \times 11088 = a + b^{(13)}$
 8) $a = 8 \times 11 \times 29 \times 107 \times 41749$, $b = 8 \times 419 \times 2129 \times 18143$,
 $\sigma(a) = 15 \times 12 \times 30 \times 108 \times 417480 = \sigma(b) = 15 \times 420 \times 2130 \times 18144 = a + b^{(13)}$
 9) $a = 8 \times 11 \times 47 \times 563 \times 49031$, $b = 8 \times 47 \times 10151 \times 32687$,
 $\sigma(a) = 15 \times 12 \times 48 \times 564 \times 49032 = \sigma(b) = 15 \times 48 \times 10152 \times 32688 = a + b^{(13)}$
 10) $a = 128 \times 149 \times 1567 \times 3923$, $b = 128 \times 3919 \times 235439$,
 $\sigma(a) = 225 \times 150 \times 1568 \times 3924 = \sigma(b) = 255 \times 3920 \times 235440 = a + b^{(13)}$
 11) $a = 64 \times 151 \times 251 \times 49559$, $b = 64 \times 227 \times 2477 \times 2478 \times 3360 = a + b$,
 $\sigma(a) = 127 \times 152 \times 252 \times 49560 = \sigma(b) = 127 \times 228 \times 2478 \times 3360 = a + b^{(13)}$
 12) $a = 64 \times 101 \times 1103 \times 17483$, $b = 64 \times 281 \times 827 \times 8431$,
 $\sigma(a) = 127 \times 102 \times 1104 \times 17484 = \sigma(b) = 127 \times 282 \times 828 \times 8432 = a + b^{(13)}$
 13) $a = 64 \times 71 \times 619 \times 44843$, $b = 64 \times 8369 \times 239167$,
 $\sigma(a) = 127 \times 72 \times 620 \times 44844 = \sigma(b) = 127 \times 8370 \times 239168 = a + b^{(13)}$
 14) $a = 64 \times 79 \times 2837 \times 8963$, $b = 64 \times 659 \times 1493 \times 2063$,
 $\sigma(a) = 127 \times 80 \times 2838 \times 8964 = \sigma(b) = 127 \times 660 \times 1494 \times 2064 = a + b^{(13)}$
 15) $a = 64 \times 163 \times 431 \times 28687$, $b = 64 \times 163 \times 977 \times 12671$,
 $\sigma(a) = 127 \times 164 \times 432 \times 28688 = \sigma(b) = 127 \times 164 \times 978 \times 12672 = a + b^{(13)}$

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The Integral Sum Number of Caterpillars

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Abstract: Given a graph $G = (V, E)$, If there exists a set $S \subset V$ such that $uv \in E$ ($u, v \in V$) if and only if $u + v \in S$, then G is $\sum$ -graph (Integral sum graph). The $\sum$ -number (Integral sum number) $\xi(G) = \min \{ |G \cup K_1| \mid G \cup K_1 \text{ is Integral sum graph} \}$. This concept is introduced by [1]. This paper proved: all caterpillars is $\sum$ -graphs, and the conjecture "all tree T that satisfy $\xi(T) = 0$ is caterpillars" is false.

Key words: Tree; Caterpillars; $\sum$ -graph; $\sum$ -number

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15 Friendly Numbers

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Abstract: In this paper, we gave 15 friendly numbers.

Key words: number theory; friendly number; complete number