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$L\aleph_1$-不分明化邻近结构和滤子

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摘要:本文基于连续值逻辑用语义方法定义了不分明化δ-邻近结构和不分明化滤子,并由不分明化δ-邻近结构给出了δ-不分明化邻域系的概念;证明了这种定义下的δ-不分明化邻域系构成不分明化滤子,得到了不分明化δ-邻近结构的可分离性与δ-不分明化邻域系N_x的T_1、T_0-可分离性的等价性。

关 键 词:不分明化δ-邻近结构;不分明化滤子;δ-不分明化邻域系

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定义1[1] 设X是一个论域,称一元F-谓词$K\in\mathscr{F}(\mathscr{F}(X))$是$X$的一个不分明化滤子,若

$$(1)\models\phi\notin K\wedge K\neq\phi$$

$(2)\models(\forall A、B)((A\in K)\wedge(B\in K)\rightarrow A\cap B\in K)$

$(3)\models(\forall A、B)((A\subseteq B)\wedge(A\in K)\rightarrow B\in K)$

定义2 设X是一个论域,称二元F-谓词$\delta\in\mathscr{F}(\mathscr{F}(X)\times\mathscr{F}(X))$是$X$上的一个不分明化$\delta$-邻近结构,若

$(1)\models(A,B)\in\delta\rightarrow(B,A)\in\delta$ 此处的"$\rightarrow$"实际上可以改成"$\leftrightarrow$"

$(2)\models(A\subseteq C)\wedge(B\subseteq D)\rightarrow((A,B)\in\delta\rightarrow(C,D)\in\delta)$

$(3)\models\neg(\phi,X)\in\delta$

$(4)\models\neg(A\cap B=\phi)\rightarrow(A,B)\in\delta$

$(5)\models\neg(A,C)\in\delta\wedge\neg(B,C)\in\delta\rightarrow\neg(A\cup B,C)\in\delta$

$(6)\models\neg(A,B)\in\delta\rightarrow(\exists C\subseteq X)(\neg(A,C)\in\delta\wedge\neg(B,X-C)\in\delta)$

定理1 设$\delta\in\mathscr{F}(\mathscr{F}(X)\times\mathscr{F}(X))$是$X$上的一个不分明化$\delta$-邻近结构,定义一元$F$-谓词$N_A\in\mathscr{F}(\mathscr{P}(X))(\phi\neq A\subseteq X)$如下:$B\in N_A:=\neg(A,X-B)\in\delta,B\subseteq X$,则$N_A$(称为$A$的$\delta$-不分明化邻域系)是$X$的一个不分明化滤子。

证明:(1) $0\leqslant N_A(\phi)=1-\delta(A,X)\leqslant1-[\neg(A\cap X=\phi)]=0\Rightarrow[\neg(\phi\in N_A)]=1$

$1\geqslant N_A(X)=1-\delta(A,\phi)\geqslant1-\delta(\phi,A)\geqslant1-\delta(\phi,X)=1\Rightarrow N_A(X)=1$

(2) $\inf\limits_{B,C\subseteq X}\min(1,1-\min(N_A(B),N_A(C))+N_A(B\cap C))$

$=\inf\limits_{B,C\subseteq X}\min(1,1-\min(1-\delta(A,X-B),1-\delta(A,X-C))+1-\delta(A,X-B\cap C))$

$\geqslant\inf\limits_{B,C\subseteq X}\min(1,1-\min(1-\delta(X-B,A),1-\delta(X-C,A))+1-\delta(A,X-B\cap C))$

$\geqslant\inf\limits_{B,C\subseteq X}\min(1,1-(1-\delta((X-B)\cup(X-C),A))+1-\delta(A,X-B\cap C))$

$=\inf\limits_{B,C\subseteq X}\min(1,\delta((X-B)\cup(X-C),A)+1-\delta(A,X-B\cap C))$

$=\inf\limits_{B,C\subseteq X}\min(1,\delta(X-B)\cap C,A)+1-\delta(A,X-B\cap C))$

$\geqslant\inf\limits_{B,C\subseteq X}\min(1,\delta(A,X-B\cap C)+1-\delta(A,X-B\cap C))=1$

(3) $\inf\limits_{B\subseteq C}\min(1,1-N_A(B)+N_A(C))$

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$= \inf\limits_{B\subseteq C}\min(1,1-(1-\delta(A,X-B))+(1-\delta(A,X-C)))$

$= \inf\limits_{B\subseteq C}\min(1,1+\delta(A,X-B)-\delta(A,X-C))\geqslant 1$

定理2 设$\delta\in\mathscr{F}(\mathscr{P}(X)\times\mathscr{P}(X))$是$X$上的一个不分明化$\delta$－邻近结构，对于$x\in A$，定义$x$的$\delta$－不分明化邻域系$N_x\in\mathscr{F}(\mathscr{P}(X))$为：$A\in N_x:=\neg(\{x\},X-A)\in\delta,A\subseteq X$.则

(1) N_x构成X的一个不分明化滤子。特别地，有$\models(\forall x)(\forall A)(A\in N_x\rightarrow x\in A)$.

(2) $\models(\forall x\neq y)(\neg(\{x\},\{y\})\in\delta$

$\leftrightarrow(\exists A)((A\in N_x)\wedge(y\notin A))\wedge(\exists B)((B\in N_y)\wedge(x\notin B)))$

(3) $\models(\forall x\neq y)(\neg(\{x\},\{y\})\in\delta$

$\leftrightarrow(\exists A)(((A\in N_x)\wedge(y\notin A))\vee((A\in N_y)\vee(x\notin A))))$.

证明：(1) 关于N_x构成X的不分明化滤子只要在定理1的证明当中取$A=\{x\}$即可。

$\because 1\geqslant \inf\limits_{x\notin A}\min(1,1-N_x(A))=\inf\limits_{x\notin A}\min(1,\delta(\{x\},X-A))$

$\geqslant \inf\limits_{x\notin A}\min(1,1-[\{x\}\cap(X-A)=\phi])=1$

$\therefore \inf\limits_{x\notin A}\min(1,1-N_x(A))=1$.

(2) 往证：$\inf\limits_{x\neq y}(1-\delta(\{x\},\{y\}))=\inf\limits_{x\neq y}\min(\sup\limits_{A\subseteq X,y\notin A}N_x(A),\sup\limits_{B\subseteq X,x\notin B}N_y(B))$

事实上，一方面

$$\inf_{x\neq y}(1-\delta(\{x\},\{y\}))=\inf_{x\neq y}N_x(X-\{y\})\leqslant\inf_{x\neq y}\sup_{A\subseteq X,y\notin A}N_x(A)$$

$$\inf_{x\neq y}(1-\delta(\{x\},\{y\}))=\inf_{x\neq y}N_y(X-\{x\})\leqslant\inf_{x\neq y}\sup_{B\subseteq X,x\notin B}N_y(B)$$

从而

$$\inf_{x\neq y}(1-\delta(\{x\},\{y\}))\leqslant\inf_{x\neq y}\min(\sup_{A\subseteq X,y\notin A}N_x(A),\sup_{B\subseteq X,x\notin B}N_y(B))$$

另一方面，设$\inf\limits_{x\neq y}\min(\sup\limits_{A\subseteq X,y\notin A}N_x(A),\sup\limits_{B\subseteq X,x\notin B}N_y(B))>\alpha,0\leqslant\alpha<1$

则对$\forall x、y\in X,x\neq y,\exists A、B\subseteq X,y\notin A,x\notin B,N_x(A)>\alpha,N_y(B)>\alpha$

所以

$$N_x(X-\{y\})=1-\delta(\{x\},\{y\})\geqslant 1-\delta(\{x\},X-A)=N_x(A)>\alpha$$

$$N_y(X-\{x\})=1-\delta(\{x\},\{y\})\geqslant 1-\delta(\{y\},X-B)=N_y(A)>\alpha$$

从而

$$\inf_{x\neq y}(1-\delta(\{x\},\{y\}))\geqslant\inf_{x\neq y}\min(\sup_{A\subseteq X,y\notin A}N_y(A),\sup_{B\subseteq X,x\notin B}N_y(B))$$

综上所述

$$\inf_{x\neq y}(1-\delta(\{x\},\{y\}))=\inf_{x\neq y}\min(\sup_{A\subseteq X,y\notin A}N_x(A),\sup_{B\subseteq X,x\notin B}N_y(B)).$$

(3) 与(2)类似可证：$\inf\limits_{x\neq y}(1-\delta(\{x\},\{y\}))=\inf\limits_{x\neq y}\max(\sup\limits_{A\subseteq X,y\notin A}N_x(A),\sup\limits_{B\subseteq X,x\notin B}N_y(B))$.

注：定理2的(2)和(3)实际上刻画了X上的不分明化δ－邻近结构的可分离性与$x(x\in X)$的δ－不分明化邻域系N_x的T_1、T_0－可分离性是等价的。

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$L_{\aleph_1}$ - Fuzzifying Proximity Structure and Filter

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Abstract: This article defines the fuzzifying δ-proximity structure and fuzzifying filter based on the continous value logic with the semantic method, and gives the concept of the δ-fuzzifying neighborhood system by the fuzzifying δ-proximity structure. It proves the δ-fuzzifying neighborhood system is the fuzzifying filter, obtains the fuzzifying δ-proximity structure separability and T_1、T_0- separability equivalence of the δ-fuzzifying neighborhood system.

Key words: fuzzifying δ-proximity structure; fuzzifying filter; δ-fuzzifying neighborhood system

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A Research on Comprehensive Evaluation for HOV Lane Based on Projection Pursuit

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Abstract: As an important basis for practical application, the comprehensive evalution for the results of HOV lanes is attracting more and more attention. To make objective, scientific and quantitative evaluation, based on the comprehensive evalution system for HOV lanes and the principles of projection pursuit, the paper presents a new method, HOV lanes comprehensive evaluation. The paper forecasts and analyzes the result of HOV lanes to provide reference for the applicayion of HOV Lanes in China.

Key words: HOV Lanes; projetion pursuit; comprehensive eualuation

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