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具轴对称初始缺陷的正交各向异性夹层圆柱薄壳的非线性稳定性

童谷生

(土木工程系)

摘　要　对具有初始缺陷的正交各向异性表层圆柱薄壳在轴压下的非线性稳定性进行了分析、研究,导出了稳定性控制方程,求得了数值解,并对屈曲载荷、初挠度、夹心模量、缺陷波数参量间的关系进行了讨论.

关键词　夹层壳;几何非线性;屈曲

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0　引　言

夹层圆柱薄壳具有很高的结构效能,是航空、航天结构中广泛采用的结构形式.文献[1]、[10]研究了各向同性表层夹层圆柱以及复合材料圆柱曲板在轴压下的非线性弹性稳定问题.文献[2]研究了复合材料多层圆柱扁壳在轴压下的后屈曲特性.文献[3]使用能量法对正交各向异性表层圆柱扁壳进行了非线性稳定性分析.文献[4]对具初始缺陷的各向同性表层夹层圆柱壳的稳定性进行了分析.本文是在文献[4]的基础上,首次对具轴对称初始缺陷的正交各向异性表层夹层圆柱薄壳采用静力法进行了几何非线性稳定性分析.采用 stein 的前屈曲一致理论得出了前屈曲阶段挠度参数随轴向载荷及缺陷参数的变化情况;采用伽辽金法得到了屈曲控制方程,并进行了数值计算以及结果讨论.

1　假　定

(1)　圆柱两表层由同一种正交各向异性材料组成,夹心是各向同性的;

(2)　表层和夹心厚度均匀,不考虑层脱和屈部屈曲;

(3)　壳体足够长可以忽略端部边界条件;

(4)　夹心层由软而轻的各向同性材料组成,略去夹心内沿 $x-y$ 面内的应力,即 $\sigma_x=\sigma_y=\tau_{xy}=0$,但 τ_{xz}, τ_{yz} 存在,即夹心可以承剪.

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2　屈曲控制方程的建立

如图 1 所示，设完善壳的中面为参考面，建立直角坐标系⒀坐标轴 x, y, z 的方向分别表示相对于参考面的轴向，周向和径向⒀壳体上点(x, y, z)处的位移 $\vec{u}^{*}=(u_1, v_1, w_1)$ 与中面上点(x, y)处的位移 $\vec{u}=(u, v, w)$ 之间有如下关系：$u_1=u-z\beta_x$，　$v_1=v-z\beta_y$，　$w_1=w$⒀其中 β_x，β_y 为中面法线转角⒀

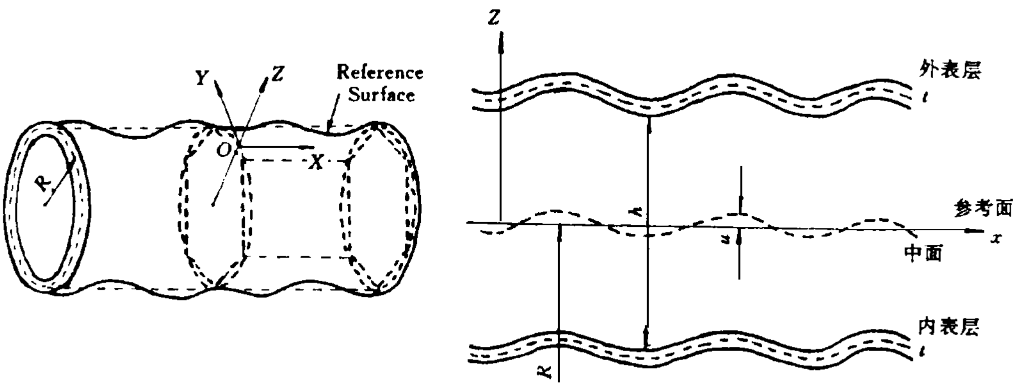

图 1　具初始缺陷的柱壳

把 Karman-Donnell 几何关系应用到柱壳上可得(x, y, z)处的应变如下

$$\begin{cases} \varepsilon_x=\varepsilon_x^0-z\dfrac{\partial\beta_x}{\partial x}, \quad \varepsilon_y=\varepsilon_y^0-z\dfrac{\partial\beta_y}{\partial y}, \quad \varepsilon_z=0, \\ \gamma_{xy}=\gamma_{xy}^0-z\left(\dfrac{\partial\beta_y}{\partial x}+\dfrac{\partial\beta_x}{\partial y}\right), \quad \gamma_{yz}=\gamma_{yz}^0, \quad \gamma_{xz}=\gamma_{xz}^0 \end{cases} \tag{1}$$

其中 ε_{ij}^0 为中面应变

$$\begin{cases} \varepsilon_x^0=\dfrac{\partial u}{\partial x}+\dfrac{1}{2}\left(\dfrac{\partial w}{\partial x}\right)^2+\dfrac{\partial w}{\partial x}\dfrac{\partial w_0}{\partial x}, \\ \varepsilon_y^0=\dfrac{\partial v}{\partial y}+\dfrac{w}{R}+\dfrac{1}{2}\left(\dfrac{\partial w}{\partial y}\right)^2+\dfrac{\partial w}{\partial y}\dfrac{\partial w_0}{\partial y}, \\ \gamma_{xy}^0=\dfrac{\partial v}{\partial x}+\dfrac{\partial u}{\partial y}+\dfrac{\partial w}{\partial x}\dfrac{\partial w}{\partial y}+\dfrac{\partial w}{\partial x}\dfrac{\partial w_0}{\partial y}+\dfrac{\partial w_0}{\partial x}\dfrac{\partial w}{\partial y}, \\ \gamma_{yz}^0=\dfrac{\partial w}{\partial y}-\beta_y, \\ \gamma_{xz}^0=\dfrac{\partial w}{\partial x}-\beta_x ⒀ \end{cases} \tag{2}$$

其中 w_0 为无应力初始挠度⒀

由几何关系代入物理方程并积分可得内力素如下

薄膜力

$$\begin{cases} N_x = A_{xx}\varepsilon_x^0 + A_{xy}\varepsilon_y^0, \\ N_y = A_{yx}\varepsilon_x^0 + A_{yy}\varepsilon_y^0, \\ N_{xy} = G_{xy}\gamma_{xy}^0 . \end{cases} \tag{3}$$

合力矩

$$\begin{cases} M_x = -D\left(A_{xx}\dfrac{\partial\beta_x}{\partial x} + A_{xy}\dfrac{\partial\beta_y}{\partial y}\right), \\ M_y = -D\left(A_{yx}\dfrac{\partial\beta_x}{\partial x} + A_{yy}\dfrac{\partial\beta_y}{\partial y}\right), \\ M_{xy} = -G_{xy}\left(\dfrac{\partial\beta_x}{\partial y} + \dfrac{\partial\beta_y}{\partial x}\right). \end{cases} \tag{4}$$

合剪力

$$\begin{cases} Q_x = hG^c\left(\dfrac{\partial w}{\partial x} - \beta_x\right), \\ Q_y = hG^c\left(\dfrac{\partial w}{\partial y} - \beta_y\right). \end{cases} \tag{5}$$

其中　$A_{ij} = \dfrac{2tE_i}{1-\gamma_i\gamma_j}$,　$G_{xy} = 2tG_{xy}$,　$D = \dfrac{1}{4}\left(h^2 + \dfrac{t^3}{3}\right)$.

$G_{xy}, E_i, \gamma_i (i=1,2)$ 分别表示表层弹性模量与泊松比,G^c 表示夹心剪切模量.

将以上内力素代入众所周知的静力平衡方程并整理得下列控制方程组

$$\begin{cases} \beta_x - \dfrac{\partial w}{\partial x} = \dfrac{D}{G^c h}\left(A_{xx}\dfrac{\partial^2\beta_x}{\partial x^2} + A_{xy}\dfrac{\partial^2\beta_x}{\partial y^2} + G_{xy}\dfrac{\partial^2\beta_y}{\partial x\partial y}\right); \\ \beta_y - \dfrac{\partial w}{\partial y} = \dfrac{D}{G^c h}\left(A_{yx}\dfrac{\partial^2\beta_y}{\partial y^2} + A_{yy}\dfrac{\partial^2\beta_y}{\partial x^2} + G_{xy}\dfrac{\partial^2\beta_x}{\partial x\partial y}\right); \\ D\left(A_{xx}\dfrac{\partial^3\beta_x}{\partial x^3} + 2G_{xy}\dfrac{\partial^3\beta_x}{\partial x\partial y^2} + 2G_{xy}\dfrac{\partial^3\beta_y}{\partial x^2\partial y} + A_{yy}\dfrac{\partial^3\beta_y}{\partial y^3}\right) = \\ \dfrac{\partial^2 F}{\partial y^2}\left(\dfrac{\partial^2 w}{\partial x^2} + \dfrac{\partial^2 w_0}{\partial x^2}\right) - 2\dfrac{\partial^2 F}{\partial x\partial y}\left(\dfrac{\partial^2 w}{\partial x\partial y} + \dfrac{\partial^2 w_0}{\partial x\partial y}\right) + \dfrac{\partial^2 F}{\partial x^2}\left(\dfrac{\partial^2 w}{\partial y^2} + \dfrac{\partial^2 w_0}{\partial y^2} - \dfrac{1}{R}\right). \\ \dfrac{1}{D_1}\left[A_{xx}\dfrac{\partial^4 F}{\partial x^4} + \left(\dfrac{D_1}{G_{xy}} - 2A_{xy}\right)\dfrac{\partial^4 F}{\partial x^2\partial y^2} + A_{yy}\dfrac{\partial^4 F}{\partial y^4}\right] = \\ \dfrac{1}{R}\dfrac{\partial^2 w}{\partial x^2} + \left(\dfrac{\partial^2 w}{\partial x\partial y}\right)^2 - \dfrac{\partial^2 w}{\partial x^2}\dfrac{\partial^2 w}{\partial y^2} - \left(\dfrac{\partial^2 w_0}{\partial x^2}\dfrac{\partial^2 w}{\partial y^2} - 2\dfrac{\partial^2 w}{\partial x\partial y}\dfrac{\partial^2 w_0}{\partial x\partial y} + \dfrac{\partial^2 w}{\partial x^2}\dfrac{\partial^2 w_0}{\partial y^2}\right). \end{cases} \tag{6}$$

其中　$D_1 = A_{xx}A_{yy} - A_{xy}^2$;　F 为 Airy 应力函数;　最后一个方程为应变协调方程,前三个为平衡方程.

3　屈曲分析

3.1　前屈曲分析

设初始缺陷可表示成

$$w_0(x) = \mu\cos\frac{2\pi x}{L_x}. \tag{7}$$

由于载荷(轴压)是轴对称的,由 stein 的前屈曲一致理论,可设前屈曲挠度函数为

$$w(x,y)=w^{*}(x)。$$

于是有对称解

$$\beta_x(x,y)=\beta_x^{*}(x),\qquad \beta_y(x,y)=0。$$

可设 Airy 应力函数为

$$F(x,y)=-\frac{1}{2}N_0y^2+F^{*}(x),$$

把以上函数代入控制方程可得如下特解

$$\begin{cases} F^{*}(x)=N\cos\dfrac{2\pi x}{L_x},\quad w^{*}(x)=K+M\cos\dfrac{2\pi x}{L_x},\\ \beta_x^{*}=P\sin\dfrac{2\pi x}{L_x}。\end{cases}\tag{8}$$

其中 M,N 可用无量纲参数表示为

$$\begin{cases} M=\lambda\mu/(\lambda_0-\lambda),\\ N=\lambda\alpha\mu/2\rho^2E_xE_y^{-1}(\lambda_0-\lambda)。\end{cases}\tag{9}$$

式中 $\alpha^2=E_xh^2t^2/(1-\nu_x\nu_y),\quad \lambda=N^0R/2\alpha$

$\gamma^2=4(1-\nu_x\nu_y)/h^2,\quad x=\alpha/G^chR,$

$L^2=4\pi^2R^2/\gamma^2,\quad \rho^2=L^2/2L_x^2,$

$\lambda_0=\dfrac{\rho^2}{1+2x\rho^2}+\dfrac{1}{4\rho^2}\dfrac{E_y}{E_x}。$

下面通过式(9)来分析前屈曲状态下挠度幅值与载荷参量以及夹心剪切模量参数 x 与波长参数之间的关系。

(1) 完善壳的轴对称屈曲

对于完善壳,因有 $\mu=0$ 和 $M\neq0$,式(8)所描述的挠度函数相应于完善壳的前屈曲模。分析可得完善壳轴对称屈曲载荷参数

$$\lambda_0=\begin{cases}\dfrac{1}{2x},\quad x>\sqrt{\dfrac{E_x}{E_y}},\\ \sqrt{\dfrac{E_y}{E_x}}-\dfrac{1}{2}\dfrac{E_y}{E_x}x,\quad x<\sqrt{\dfrac{E_x}{E_y}}。\end{cases}\tag{10}$$

(2) 前屈曲挠度的增长

从解的表达式可以看出:挠度参数 M 随着载荷的增加而增加;对于给定的外载,具有轴对称屈曲模形式的缺陷将产生最大前屈曲挠度;$\dfrac{M}{\mu}$ 随 ρ^2 的关系如图 2 所示。

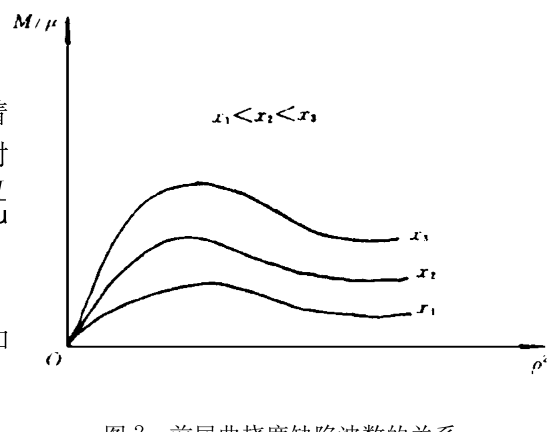

图 2 前屈曲挠度缺陷波数的关系

3.2 分叉屈曲解

设分叉处增量为 $w(x,y)$,$f(x,y)$ 和 $b_x(x,y)$,以及 $b_y(x,y)$,由此得分叉点附近的解

$$
\begin{cases}
F(x,y) = -0.5N_0y^2 + F^*(x) + f(x,y), \\
w(x,y) = w^*(x) + w(x,y), \\
\beta_x(x,y) = \beta^*(x) + b_x(x,y), \\
\beta_y(x,y) = b_y(x,y)。
\end{cases}
\tag{11}
$$

将式(11)代入控制方程组(6)便可求出后屈曲解所应满足的微分方程组。我们在假定模函数形式下采用伽辽金法可导出屈曲问题的特征方程,并可进行数值计算。

4　数值结果与讨论

4.1　一级近似解

取模函数 $w = K\cos\frac{\pi x}{L_x}\cos\frac{\pi y}{L_y}$ 代入控制方程,可得 b_x、b_y、f 的特解,且得到特征方程如下

$$\lambda^3 + p_1\lambda^2 + p_2\lambda + p_3 = 0。\tag{12}$$

其中

$$p_1 = \frac{k_0}{4\alpha\rho^2} - E_1\rho^2 - 2\lambda_0 + \frac{\gamma\mu\tau^2 E_y}{4\rho^2 E_x},$$

$$p_2 = \lambda_0^2 + 2E_1\rho^2\lambda_0 - \frac{k_0\lambda_0}{2\alpha\rho^2} - \frac{\lambda_0\mu\gamma\tau^2}{4\rho^2}\frac{E_y}{E_x},$$

$$p_3 = \frac{k_0\lambda_0^2}{4\alpha\rho^2} - E_1\lambda_0^2\rho^2 - 2\mu\gamma\rho^2\tau^2E_1\lambda_0 - \gamma^2\mu^2\lambda_0\rho^2\tau^2(E_1 + E_2)。$$

且

$$\tau^2 = Rh\pi^2/L_x^2\sqrt{1-\gamma_x\gamma_y},$$

$$E_1 = E_y/\{E_x[\rho^4 + (E_y/G_{xy} - 2\gamma_y)\rho^2\tau^2 + E_y\tau^4/E_x]\},$$

$$E_2 = E_y/\{E_x[81\rho^4 + 9(E_y/G_{xy} - 2\gamma_y)\rho^2\tau^2 + E_y\tau^4/E_x]\}。$$

4.2　非完善壳的屈曲载荷参数的数值计算

设夹层柱壳表层为正交各向异性的玻璃环氧(glass-expory)材料,夹心为泡沫且

$$\frac{E_x}{E_y} = 3.0,\quad \frac{G_{xy}}{E_y} = 0.60,\quad \gamma_x = 0.25,\quad t = 1(\text{个单位})$$

$$h = 10t,\quad x = 0.25, 0.5, 0.75$$

同时选取缺陷参数 $\mu_k = \frac{\mu}{t}$,μ_k 取值如下

$$\mu_k = 0.05, 0.2, 0.3, 0.5, 0.6, 0.7, 0.8,$$

数值结果见表 1—3。

4.3　结果讨论

数值计算表明,对于具轴对称缺陷的非完善壳有以下几个结论。

(1)　对于给定的缺陷参数,分叉载荷随着波数 ρ 的增加开始下降,随后随着 ρ 的增大而较快增加,这表明与实际屈曲模态相同的初始缺陷有很大的危害性;

(2)　分叉载荷随着夹心的柔度参数 x 的增大而降低,这表明可以通过优化夹心以提高屈曲载荷;

(3)　与文献[4]的结论相比较表明,只要提高壳体表层沿轴向的抗压刚度(E_x),可相对提

高屈曲荷载(13)

表 1　初始缺陷参数$\frac{\mu}{t}$对屈曲荷载的影响　$x=0.25$

ρ / μ/t	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0
0.05	3.0828	2.7684	2.6750	2.6591	2.7045	2.8223	3.0411	3.3988	3.9361	4.6715
0.20	3.0709	2.7660	2.6741	2.6587	2.7043	2.8223	3.0409	3.3996	3.9358	4.6712
0.30	3.0604	2.7640	2.6734	2.6584	2.7042	2.8222	3.0408	3.3995	3.9356	4.6710
0.50	3.0515	2.7622	2.6728	2.6581	2.7041	2.8222	3.0407	3.3992	3.9353	4.6706
0.60	3.0384	2.7597	2.6718	2.6577	2.7039	2.8222	3.0407	3.3991	3.9351	4.6705
0.70	3.0343	2.7590	2.6716	2.6576	2.7039	2.8222	3.0406	3.3990	3.9350	4.6703
0.80	3.0313	2.7585	2.6715	2.6575	2.7040	3.0406	3.0406	3.3989	3.9348	4.6701

表 2　初始缺陷参数$\frac{\mu}{t}$对屈曲荷载的影响　$x=0.5$

ρ / μ/t	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0
0.05	3.0364	2.6992	2.5879	2.5536	2.5739	2.6503	2.7959	3.0353	3.3947	3.8974
0.20	3.0252	2.6971	2.5872	2.5533	2.5738	2.6502	2.7959	3.0351	3.3944	3.8971
0.30	3.0154	2.6953	2.5866	2.5531	2.5737	2.6501	2.7958	3.0349	3.3943	3.8970
0.50	3.0071	2.6935	2.5861	2.5529	2.5736	2.6501	2.7956	3.0347	3.3940	3.8966
0.60	2.9948	2.6917	2.5854	2.5526	2.5736	2.6501	2.7956	3.0346	3.3938	3.8964
0.70	2.9910	2.6910	2.5852	2.5526	2.5735	2.6501	2.7955	3.0345	3.3937	3.8962
0.80	2.9882	2.6906	2.5851	2.5525	2.5735	2.6501	2.7955	3.044	3.3936	3.8961

表 3　初始缺陷参数$\frac{\mu}{t}$对屈曲荷载的影响　$x=0.75$

ρ / μ/t	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0
0.05	3.0315	2.5867	2.5610	2.5317	2.5713	2.5930	2.6017	2.7312	3.047	3.3576
0.20	3.0147	2.5862	2.5607	2.5312	2.5709	2.5924	2.6013	2.7307	3.043	3.3573
0.30	3.001	2.5857	2.5602	2.5308	2.5706	2.5921	2.6010	2.7305	3.040	3.3569
0.50	2.9847	2.5852	2.5548	2.5302	2.5702	2.5918	2.6008	2.7301	3.037	3.3566
0.60	2.9763	2.5849	2.5542	2.5290	2.5700	2.5916	2.6005	2.7296	3.036	3.3562
0.70	2.9610	2.5846	2.5537	2.5288	2.5696	2.5913	2.6002	2.7292	3.033	3.3560
0.80	2.9513	2.5843	2.5533	2.5283	2.5693	2.5910	2.5917	2.7290	3.030	3.3557

参 考 文 献

1 中国科学院力学研究所19.夹层板壳的弯曲、稳定和振动19.北京:科学出版社,1977 19.180～189

2 Cha Chuenyuan· Non-Linear Free Vibration and Post Buckling of Symmetrically Laminated Orthotropic Imperfect Shallow Cylindical Panes with Two Adjacent Edges Simply supported and the other Edges Champed· Int· J Solids Strctures, 1987, 23(8):1123～1132

3 王颖坚等19.正交各向异性表层的夹层圆柱扁壳的非线性稳定性19.应用数学和力学, 1989, 10(2)

4 R· C· Tennyson, K· C· Chen· Buckling of Imperfect Sandwich Cylinders Under Axial Compression· Int· J· Solids Structures, 1990, 126(9/10): 1017～1036

5 王震鸣等19.正交各向异性多层夹层和加筋扁壳的弯曲、稳定和振动19.力学学报,1983

6 范钦珊等19.圆柱壳的初始后屈曲及对局部缺陷的敏感性19.清华大学学报, 1990, 30

7 周承倜等19.弹性稳定理论19.应用数学和力学讲座丛书19.成都:四川人民出版社,1981

8 徐芝纶19.弹性理论(下册) 19.北京:人民教育出版社,1977

9 工程材料实用手册编辑委员会19.工程材料实用手册19.北京:科学出版社,1984

10 王震鸣19.复合材料力学和复合材料结构力学19.北京:机械工业出版社,1991

Non-Linear Buckling of Imperfect Sandwich Cylinders with Orthotropic Faces under Axial Compression

Tong Gusheng

(Civil Engineering Department)

Abstract　An analytical study has been carried out to determine the effect of axisymmetric shape imperfection on the compression buckling strength of sandwich cylinders which have orthotropic faces and isotropic, shear deformable cores· Then a buckling solution is presented as a function of imperfection amplitude, wave-length and core shear flexibility coeffients, Finally, a numerical example and its related results are discussed·

Key words　sandwich cylinders; geometric non-liner; buckling