

随机高斯干扰时 MOK 通信方式的误码率计算

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摘要 分析非相干 M 进制正交键控信号在随机高斯干扰信道中的平均差错概率;给出了计算非相干 M 进制正交键控信号在包络平方检波时,平均差错概率的通用表达式,为分析 M 进制正交键控信号提供了数学工具.

关键词 M 进制正交键控;调制/解调;非相干接收;差错概率

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0 概述

在 M 进制正交键控(M-ary Orthogonal Keyed:MOK)信号通信系统中, M 个相互正交的信号用于数据传送,每符号周期内传送 $\log_2 M$ 比特信息.信息检测可以通过相关匹配滤波器或者有源相关器来实现.图1所示为一种非相干 MOK 接收机,它有 M 条接收支路,每条支路包括两个匹配滤波器和两个平方检波器用于接收同向支路和正交支路的信息.同向支路和正交支路相加组合产生判决变量.有关 M 进制正交信号通信方式的研究,已经有许多研究结果并且得到了广泛应用^[1~3].但是这些研究都基于假设:干扰信号是一宽带白色高斯噪声(Broad-band white Gaussian noise)^[9].

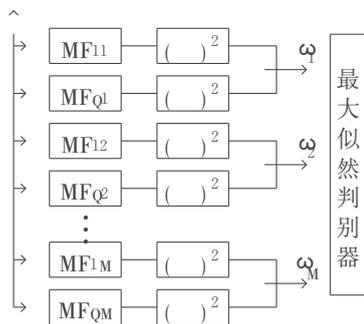


图1 M进制正交信号通信系统非相干接收机

但是,在一些通信环境中,干扰信号服从随机高斯分布,不服从宽带白色高斯信道模型.本文给出的结果是对 MOK 通信系统理论的一个补充.

1 性能分析

这里描述的 M 进制正交键控(M-ary Orthogonal Keying:MOK)系统所发送的信号 $s_i(t)$, $i = 0, 1, \dots, M - 1$,是 M 个等能量的相互正交信号.非相干 M 进制正交键控系统接收机如图1所示,接收信号首先经过匹配滤波器,产生 M 个正交分量和 M 个同相分量,然后各支路

的正交分量和同相分量平方相加,得出各支路的判决变量.最大似然判别器从 M 个判决变量中选择最大的一个映射成输出数据.

设接收机同参考信号完全同步,并且假设进入接收机的干扰信号在匹配相关之后是一均值为零的高斯随机过程,但是各支路的高斯随机过程的功率谱密度可以不等.设发送信号(即参考信号)为 $s_{\mu}(t)$,则相应接收支路归一化输出信号可表示为

$$\omega_{\mu} = 1 + n_{\mu}, \quad (1)$$

其中 n_{μ} 是噪声项(由白噪声和干扰产生)^[13]由于各信号之间正交,其它接收支路输出只有噪声,因此,输出信号为

$$\omega = n_i, \quad i = 0, 1, \dots, M - 1; i \neq \mu \quad (2)$$

由于采用了平方检波器,所以 ω_{μ} 为平方分布,其概率密度函数为

$$f_{\omega_{\mu}}(\omega_{\mu}) = \frac{1}{2N_{\mu}} \exp\left[-\frac{1 + \omega_{\mu}}{2N_{\mu}}\right] I_0\left(\frac{\omega_{\mu}}{N_{\mu}}\right), \quad \omega_{\mu} \geq 0 \quad (3)$$

其中 $I_0(x)$ 是零阶改进型贝赛尔函数, ω 为指数分布,概率密度函数为

$$f_{\omega}(\omega) = \frac{1}{2N_i} \exp\left[-\frac{\omega}{2N_i}\right], \quad \omega \geq 0 \quad (4)$$

在式(3)和式(4)中, N_{μ} 和 N_i 为归一化噪声功率^[13]设 $\omega_i (i = 0, 1, 2, \dots, M - 1)$ 统计独立,这一假设在许多实际系统中均近似满足,所以输出判决量的联合概率密度函数可以表示为

$$f_{\omega_0, \omega_1, \dots, \omega_{M-1}}(\omega_0, \dots, \omega_{M-1}) = \prod_{i=0}^{M-1} f_{\omega_i}(\omega_i), \quad (5)$$

根据最大似然判别原则,在发送符号 $S_{\mu}(t)$ 条件下,平均符号差错概率

$$P_{s/\mu} = 1 - \text{Prob}\left[\omega_{\mu} > \omega \mid \begin{matrix} i = 0, 1, \dots, M - 1 \\ i \neq \mu \end{matrix}\right] = \\ 1 - \int_0^{\infty} f_{\omega_{\mu}}(\omega_{\mu}) \left[\prod_{\substack{i=0 \\ i \neq \mu}}^{M-1} \int_0^{\omega_{\mu}} f_{\omega_i}(\omega) d\omega \right] d\omega_{\mu}, \quad (6)$$

由式(4)可知

$$\int_0^{\omega_{\mu}} f_{\omega_i}(\omega) d\omega = 1 - \exp\left[-\frac{\omega_{\mu}}{2N_i}\right], \quad (7)$$

设 $x = \omega_{\mu}/N_{\mu}$, $\alpha = 1/N_{\mu}$ 并将式(7)代入式(6)化简得

$$P_{s/\mu} = \int_0^{\infty} x I_0(\alpha x) \exp\left[-\frac{x^2 + \alpha}{2}\right] \left\{ 1 - \prod_{\substack{i=0 \\ i \neq \mu}}^{M-1} \left[1 - \exp\left[-\frac{x^2}{2} \cdot \frac{N_{\mu}}{N_i}\right] \right] \right\} dx, \quad (8)$$

设

$$F = 1 - \prod_{\substack{i=0 \\ i \neq \mu}}^{M-1} \left[1 - \exp\left[-\frac{x^2}{2} \cdot \frac{N_{\mu}}{N_i}\right] \right], \quad (9)$$

其中 $N_{\mu}/N_i > 0$,通过连乘和重新整理可得

$$F = \sum_{k=1}^{M-1} \sum_{n=1}^{\binom{M-1}{k}} (-1)^{k+1} \exp\left[-\frac{x^2}{2} \cdot N_{\mu} \sum_{i \in C_{nk}^M} \frac{1}{N_i}\right], \quad (10)$$

式(10)中, C_{nk}^M 按下面方法求得^[13]

$$C_{nk}^M = \left\{ u \mid \begin{matrix} u = 0, 1, \dots, M - 1 \\ u \neq \mu \end{matrix} \right\}, \quad (11)$$

从 $I^{M\mu}$ 中任意取出 k 个组成一个集合,然后将从 $I^{M\mu}$ 中取得的所有不同集合组成一个集合,得

$$C \begin{pmatrix} M-1 \\ k \end{pmatrix} = \left\{ C_{1k}^{M\mu}, C_{2k}^{M\mu}, \dots, C_{\binom{M-1}{k}}^{M\mu} \right\}, \quad (12)$$

式(10)中的 $C_{nk}^{M\mu}$ 就是 $C \begin{pmatrix} M-1 \\ k \end{pmatrix}$ 的第 n 个元素.

将式(10)代入式(8),交换积分求和次序得

$$P_{s/\mu} = \sum_{k=1}^{M-1} \left(\sum_{n=1}^{\binom{M-1}{k}} (-1)^{k+1} \int_0^{\infty} x I_0(\alpha) \exp\left(-\frac{x^2 + \alpha + \beta x^2}{2}\right) dx \right), \quad (13)$$

其中 $\beta = N_{\mu} \sum_{i \in C_{nk}^{M\mu}} 1/N_i$

设 $t = x^2/2$, 并参考文献[4], 可求得

$$P_{s/\mu} = \sum_{k=1}^{M-1} \left(\sum_{n=1}^{\binom{M-1}{k}} \frac{(-1)^{k+1}}{1 + N_{\mu} \sum_{i \in C_{nk}^{M\mu}} 1/N_i} \exp\left[-\frac{1}{2} \cdot \frac{\sum_{i \in C_{nk}^{M\mu}} 1/N_i}{1 + N_{\mu} \sum_{i \in C_{nk}^{M\mu}} 1/N_i}\right] \right), \quad (14)$$

最后,平均符号差错概率

$$\bar{P}_s = \frac{1}{M} \sum_{\mu=1}^M P_{s/\mu}, \quad (15)$$

下面讨论两种特殊情况

A. 对于 BFSK, 设两路噪声功率分别为 N_{+1} 和 N_{-1} , 则当 $M = 2, \mu = 1$ 时,

$$P_{s/+1} = \frac{N_{-1}}{N_{+1} + N_{-1}} \exp\left[-\frac{1}{2(N_{+1} + N_{-1})}\right], \quad (16)$$

$$P_{s/-1} = \frac{N_{+1}}{N_{+1} + N_{-1}} \exp\left[-\frac{1}{2(N_{+1} + N_{-1})}\right], \quad (17)$$

所以

$$\bar{P}_{\text{bfsk}} = \frac{1}{2}(P_{s/+1} + P_{s/-1}) = \frac{1}{2} \exp\left[-\frac{1}{2(N_{+1} + N_{-1})}\right], \quad (18)$$

B. 当发送信号为 M 进制正交信号, 并且 $N_i = N, i = 0, 1, \dots, M-1$ 时, 则 $\sum_{i \in C_{nk}^{M\mu}} 1/N_i =$

k/N , 所以, 平均符号差错概率

$$\begin{aligned} \bar{P}_M = P_{s/\mu} &= \sum_{k=1}^{M-1} \left(\sum_{n=1}^{\binom{M-1}{k}} \frac{(-1)^{k+1}}{1 + kN/N} \exp\left[-\frac{1}{2} \frac{k/N}{1 + kN/N}\right] \right) \\ &= \sum_{k=1}^{M-1} \binom{M-1}{k} \frac{(-1)^{k+1}}{k+1} \exp\left[-\frac{k}{k+1} \cdot \frac{1}{2N}\right], \end{aligned} \quad (19)$$

上面求出的两种特殊情况下的平均差错概率都是广为人知的结果^[1,2].

2 结 论

本文分析了非相干 MOK 信号在随机高斯干扰下的差错特性, 得到的结果可以用于诸如此类系统的差错特性分析¹⁹.

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Analysis of Dynamic Optimal Design for Cam-Follower Mechanism

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Abstract In order to meet the requirement of minimum distort of follower dynamic response, the ITSE rule based on optimum theory is used to dynamic design of high speed cam-follower mechanism in this paper. The Change law of main parameters of optimally designing the mechanism is discussed. It provides a valuable basis for designing the high speed cam-follower mechanism.

Key words cam mechanism; optimal design; main parameters

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Error Probability Analysis of MOK Signals in the Presence of Arbitrary Gaussian Interference

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Abstract: A general error probability analysis for the noncoherent M-ary orthogonal keyed(MOK) signals with square envelope detector is presented in this paper. The derived results can be used for the performance evaluation of the systems with MOK modulation and square envelope detection.

Key words: M-ary orthogonal keying; modulation/demodulation; noncoherent receiver; error probability