复合材料层合板几何非线性分析

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摘　要　本文用有限条方法分析了复合材料层合板的几何非线性问题，并用 Nowton-Raphson 迭代法求解非线性方程组，计算得到的线性解与非线性解分别与经典解析解和实验解吻合较好⒆.

关键词　层合板；几何非线性；Nowton-Raphson 迭代法

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0　引　言

自从1968年 Y K Cheung 提出有限条法以来，解决了土建结构设计中大量的问题，特别是结构有一个方向比较规整的板梁结构、折板结构与箱形梁桥，板、壳的线性分析及稳定与振动分析⒀有限条选择了振动梁函数及代数多项式两种位移函数保证了计算收敛性，它形成的总刚度矩阵呈带状，与有限元法比较，它的未知量大大减少，精度也高⒀但在复合材料领域用有限条方法分析的还比较少⒀与各向同性材料相比，复合材料结构需要考虑材料各向异性、纤维方向、横向剪切、耦合效应等诸多因素，特别是几何非线性问题，因而更为复杂和困难⒀文献〔2〕用有限条法分析了各向同性材料的几何非线性问题，文献〔3〕用经典解析法与实验分别对复合材料层合板作了研究，本文在文献〔2〕的基础上考虑各向异性的复合材料层合板几何非线性问题，得到结果与文献〔3〕的实验结果比较，吻合较好⒀

1　非线性层合板理论

由冯·卡门应变得

$$\{\varepsilon\}=\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_{xy}\end{Bmatrix}=\begin{Bmatrix}\dfrac{\partial u}{\partial x}+\dfrac{1}{2}\left(\dfrac{\partial w}{\partial x}\right)^2-z\dfrac{\partial^2 w}{\partial x^2}\\ \dfrac{\partial v}{\partial y}+\dfrac{1}{2}\left(\dfrac{\partial w}{\partial y}\right)^2-z\dfrac{\partial^2 w}{\partial y^2}\\ \dfrac{\partial u}{\partial y}+\dfrac{\partial v}{\partial x}+\dfrac{\partial w}{\partial x}\dfrac{\partial w}{\partial y}-2z\dfrac{\partial^2 w}{\partial x\partial y}\end{Bmatrix}\tag{1}$$

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式中:u、v 表示中面的平面位移;w 表示中面的挠度[13]

由广义虎克定律得某一层的纤维增强层合板应力－应变关系为

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \sigma_{xy}\end{Bmatrix}^k=\begin{bmatrix}Q_{11}&Q_{12}&Q_{16}\\ Q_{12}&Q_{22}&Q_{26}\\ Q_{16}&Q_{26}&Q_{66}\end{bmatrix}^k\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_{xy}\end{Bmatrix}^k=c\{\varepsilon\} \tag{2}$$

单元体的应变能

$$牁U_p=\frac{1}{2}\sum_{k=1}^{n}\{\sigma\}^T\{\varepsilon\}=\frac{1}{2}\sum_{k=1}^{n}\{\varepsilon\}^T[c]\{\varepsilon\}=\frac{1}{2}\int_{-\frac{h}{2}}^{\frac{h}{2}}\{\varepsilon\}^T[c]\{\varepsilon\}dz$$

$$=\frac{1}{2}\left\{\begin{bmatrix}\left(\frac{\partial u}{\partial x}\right)^2 & \frac{\partial u}{\partial x}\frac{\partial v}{\partial y} & \frac{\partial u}{\partial x}\frac{\partial u}{\partial y}+\frac{\partial u}{\partial x}\frac{\partial v}{\partial y}\\ \frac{\partial u}{\partial x}\frac{\partial v}{\partial y} & \left(\frac{\partial v}{\partial y}\right)^2 & \frac{\partial u}{\partial y}\frac{\partial v}{\partial y}+\frac{\partial v}{\partial x}\frac{\partial v}{\partial y}\\ \frac{\partial u}{\partial x}\frac{\partial u}{\partial y}+\frac{\partial u}{\partial x}\frac{\partial v}{\partial y} & \frac{\partial u}{\partial y}\frac{\partial v}{\partial y}+\frac{\partial v}{\partial x}\frac{\partial v}{\partial y} & \left(\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial v}{\partial x}\right)^2+2\frac{\partial v}{\partial x}\frac{\partial u}{\partial y}\end{bmatrix}[A]+\right.$$

$$\begin{bmatrix}\frac{\partial u}{\partial x}\left(\frac{\partial w}{\partial x}\right)^2 & \frac{1}{2}\frac{\partial v}{\partial y}\left(\frac{\partial w}{\partial x}\right)^2+\frac{1}{2}\frac{\partial u}{\partial x}\left(\frac{\partial w}{\partial y}\right)^2 & \frac{\partial u}{\partial x}\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2\left(\frac{\partial u}{\partial y}\ \frac{\partial v}{\partial x}\right)\\ \frac{1}{2}\frac{\partial v}{\partial y}\left(\frac{\partial w}{\partial x}\right)^2+\frac{1}{2}\frac{\partial u}{\partial x}\left(\frac{\partial w}{\partial y}\right)^2 & \frac{\partial v}{\partial y}\left(\frac{\partial w}{\partial y}\right)^2 & \frac{\partial v}{\partial y}\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}+\frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\\ \frac{\partial u}{\partial x}\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right) & \frac{\partial v}{\partial y}\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}+\frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right) & 2\frac{\partial u}{\partial y}\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}+2\frac{\partial v}{\partial x}\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\end{bmatrix}[A]+$$

$$\begin{bmatrix}\frac{1}{4}\left(\frac{\partial w}{\partial x}\right)^4 & \frac{1}{4}\left(\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\right)^2 & \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\\ \frac{1}{4}\left(\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\right)^2 & \frac{1}{4}\left(\frac{\partial w}{\partial y}\right)^4 & \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\\ \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2\frac{\partial w}{\partial x}\frac{\partial w}{\partial y} & \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2\frac{\partial w}{\partial x}\frac{\partial w}{\partial y} & \left(\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\right)^2\end{bmatrix}[A]-$$

$$
\begin{bmatrix}
2\frac{\partial u}{\partial x}\left(\frac{\partial^2 w}{\partial x^2}\right) & \frac{\partial u}{\partial x}\frac{\partial^2 w}{\partial y^2}+\frac{\partial v}{\partial y}\frac{\partial^2 w}{\partial x^2} & 2\frac{\partial u}{\partial x}\frac{\partial^2 w}{\partial x\,\partial y}+\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\frac{\partial^2 w}{\partial x^2} \\
\frac{\partial u}{\partial x}\frac{\partial^2 w}{\partial y^2}+\frac{\partial v}{\partial y}\frac{\partial^2 w}{\partial x^2} & 2\frac{\partial u}{\partial y}\frac{\partial^2 w}{\partial y^2} & 2\frac{\partial v}{\partial y}\frac{\partial^2 w}{\partial x\,\partial y}+\left(\frac{\partial u}{\partial y}+\frac{\partial u}{\partial x}\right)\frac{\partial^2 w}{\partial y^2} \\
2\frac{\partial u}{\partial x}\frac{\partial^2 w}{\partial x\,\partial y}+\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\frac{\partial^2 w}{\partial x^2} & 2\frac{\partial v}{\partial y}\frac{\partial^2 w}{\partial x\,\partial y}+\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\frac{\partial^2 w}{\partial y^2} & 4\frac{\partial u}{\partial y}\frac{\partial^2 w}{\partial x\,\partial y}+4\frac{\partial v}{\partial x}\frac{\partial^2 w}{\partial x\,\partial y}
\end{bmatrix}[B]-
$$

$$
\begin{bmatrix}
\frac{\partial^2 w}{\partial x^2}\left(\frac{\partial w}{\partial x}\right)^2 & \frac{1}{2}\frac{\partial^2 w}{\partial x^2}\left(\frac{\partial w}{\partial y}\right)^2+\frac{1}{2}\frac{\partial^2 w}{\partial y^2}\left(\frac{\partial w}{\partial x}\right)^2 & \frac{\partial^2 w}{\partial x\,\partial y}\left(\frac{\partial w}{\partial x}\right)^2+\frac{\partial^2 w}{\partial y^2}\frac{\partial w}{\partial x}\frac{\partial w}{\partial y} \\
\frac{1}{2}\frac{\partial^2 w}{\partial x^2}\left(\frac{\partial w}{\partial y}\right)^2+\frac{1}{2}\frac{\partial^2 w}{\partial y^2}\left(\frac{\partial w}{\partial x}\right)^2 & \frac{\partial^2 w}{\partial y^2}\left(\frac{\partial w}{\partial y}\right)^2 & \frac{\partial^2 w}{\partial x\,\partial y}\left(\frac{\partial w}{\partial y}\right)^2+\frac{\partial^2 w}{\partial y^2}\frac{\partial w}{\partial x}\frac{\partial w}{\partial y} \\
\frac{\partial^2 w}{\partial x\,\partial y}\left(\frac{\partial w}{\partial x}\right)^2+\frac{\partial^2 w}{\partial x^2}\frac{\partial w}{\partial x}\frac{\partial w}{\partial y} & \frac{\partial^2 w}{\partial x\,\partial y}\left(\frac{\partial w}{\partial y}\right)^2+\frac{\partial^2 w}{\partial y^2}\frac{\partial w}{\partial x}\frac{\partial w}{\partial y} & 4\frac{\partial^2 w}{\partial x\,\partial y}\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}
\end{bmatrix}[B]+
$$

$$
\left.
\begin{bmatrix}
\left(\frac{\partial^2 w}{\partial x^2}\right)^2 & \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} & 2\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial x\,\partial y} \\
\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} & \left(\frac{\partial^2 w}{\partial y^2}\right)^2 & 2\frac{\partial^2 w}{\partial y^2}\frac{\partial^2 w}{\partial x\,\partial y} \\
2\frac{\partial^2 w}{\partial y^2}\frac{\partial^2 w}{\partial y^2} & 2\frac{\partial^2 w}{\partial y^2}\frac{\partial^2 w}{\partial x\,\partial y} & 4\left(\frac{\partial^2 w}{\partial x\,\partial y}\right)^2
\end{bmatrix}[D]\right\} \tag{3}
$$

其中

$$
A=\begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix};\quad
B=\begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix};\quad
D=\begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \tag{13}
$$

$$
A_{ij}=\sum_{k=1}^{n}\bar{Q}_{ij}(Z_k-Z_{k-1});\quad B_{ij}=\frac{1}{2}\sum_{k=1}^{n}\bar{Q}_{ij}(Z_k^2-Z_{k-1}^2);\quad D_{ij}=\frac{1}{3}\sum_{k=1}^{n}\bar{Q}_{ij}(Z_k^3-Z_{k-1}^3) \tag{13}
$$

对于多层正交各向异性对称层合板有

$$
A_{16}=A_{26}=B_{ij}=D_{16}=D_{26}=0,
$$

所以,式(3)展开并简化为

$$\triangle U_p=\frac{1}{2}\left\{\left(\frac{\partial u}{\partial x}\right)^2A_{11}+2\frac{\partial u}{\partial x}\frac{\partial u}{\partial y}A_{12}+\left(\frac{\partial v}{\partial y}\right)^2A_{22}+\left[\left(\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial u}{\partial x}\right)^2+2\frac{\partial u}{\partial x}\frac{\partial u}{\partial y}\right]A_{66}+\left(\frac{\partial^2 w}{\partial x^2}\right)^2D_{11}+2\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}D_{12}+\left(\frac{\partial^2 w}{\partial y^2}\right)^2D_{22}+4\left(\frac{\partial^2 w}{\partial x\partial y}\right)^2D_{66}\right\}+\frac{1}{2}\left\{\frac{\partial u}{\partial x}\left(\frac{\partial w}{\partial x}\right)^2A_{11}+\left[\frac{\partial v}{\partial y}\left(\frac{\partial w}{\partial x}\right)^2+\frac{\partial u}{\partial x}\left(\frac{\partial w}{\partial y}\right)^2\right]A_{12}+\frac{\partial v}{\partial y}\left(\frac{\partial w}{\partial x}\right)^2A_{22}+2\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}A_{66}\right\}+\frac{1}{2}\left\{\frac{1}{4}\left(\frac{\partial w}{\partial x}\right)^4A_{11}+\frac{1}{2}\left(\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\right)^2A_{12}+\frac{1}{4}\left(\frac{\partial w}{\partial y}\right)^4A_{22}+\left(\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\right)^2A_{66}\right\}⒀ \tag{4}$$

单元体的势能为

$$\triangle V_p=-wq_w(x,y)⒀ \tag{5}$$

2　有限条方法的公式

有限条模型为如图1所示的高阶条⒀

位移函数为

$$\{u\quad v\quad w\}^{\mathrm{T}}=\sum_{i=1}^{r}S(x)\,\Phi_n(y)A_{ni} \tag{6}$$

其中

$$S(x)=\begin{bmatrix}U_i(x)&0&0\\0&V_i(x)&0\\0&0&W_i(x)\end{bmatrix};$$

$$\Phi_n(y)=\begin{bmatrix}\varphi_n(y)&0&0\\0&\varphi_n(y)&0\\0&0&\varphi_n(y)\end{bmatrix};$$

$$\varphi_n(y)=[1\quad y\quad y^2\cdots y^n]⒀$$

对于图1的有限条模型 $n=3$

$A_{ni}=\{A_1,A_2,\cdots,A_{3n+3}\}^{\mathrm{T}}$　由条的广义自由度决定,条的广义自由度为

$$d_{ni}=\{u_1,v_1,w_1,u_2,v_2,w_2,\cdots,u_{n+1},v_{n+1},w_{n+1}\}^{\mathrm{T}}$$

其关系为

$$A_{ni}=C_nd_{ni} \tag{7}$$

C_n 的求法可参阅文献[4]⒀

把式(6)、(7)代入式(4)整理并且沿整个条积分得

$$U_p=\frac{1}{2}d_n^{\mathrm{T}}K_nd_n+\frac{1}{6}d_n^{\mathrm{T}}K_{1n}d_n+\frac{1}{12}d_n^{\mathrm{T}}K_{2n}d_n \tag{8}$$

其中

$$d_n=\{d_{n1},\ d_{n2},\ \cdots,\ d_{nr}\}^{\mathrm{T}} \tag{9}$$

$$K_n=C^{\mathrm{T}}\ S\ C \tag{10}$$

$$K_{1n}=C^{\mathrm{T}}\ S_1\ C \tag{11}$$

$$K_{2n}=C^{\mathrm{T}}\ S_2\ C \tag{12}$$

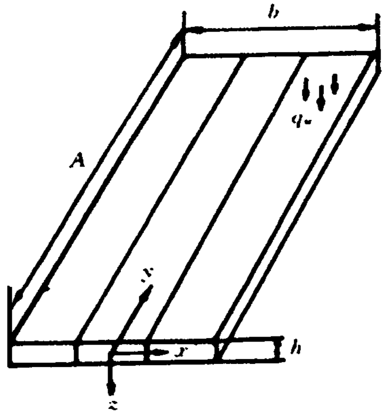

图1　有限条模型

$$S=\sum_{i=1}^{r}\sum_{j=1}^{r}X_{ij}=\begin{pmatrix} X_{11} & X_{12} & \cdots & X_{1r} \\ X_{21} & X_{22} & \cdots & X_{2r} \\ \vdots & \vdots & \vdots & \vdots \\ X_{r1} & X_{r2} & \cdots & X_{rr} \end{pmatrix}$$

$$X_{ij}=\int_{-\frac{b}{2}}^{\frac{b}{2}}\int_{0}^{A}\left(\begin{array}{c|c|c} \begin{array}{l} A_{11}U_i'U_j'\Phi_n^T\Phi_n+ \\ A_{66}U_i'U_j'\dot{\Phi}_n^T\dot{\Phi}_n \end{array} & \begin{array}{l} A_{66}U_iV_j'\dot{\Phi}_n^T\Phi_n+ \\ A_{12}U_i'V_j\Phi_n^T\dot{\Phi}_n \end{array} & 0 \\ \hline \begin{array}{l} A_{12}V_iU_j'\dot{\Phi}_n^T\Phi_n+ \\ A_{66}V_i'U_j\Phi_n^T\dot{\Phi}_n \end{array} & \begin{array}{l} A_{22}V_iV_j\dot{\Phi}_n^T\dot{\Phi}_n+ \\ A_{66}V_i'V_j'\Phi_n^T\Phi_n \end{array} & 0 \\ \hline 0 & 0 & \begin{array}{l} D_{11}W_i''W_j''\Phi_n^T\Phi_n \\ +D_{22}W_iW_j\Phi_n^T\Phi_n \\ +4D_{66}W_i'W_j'\dot{\Phi}_n\dot{\Phi}_n \\ +D_{12}W_i''W_j''\Phi_n^T\Phi_n \\ +D_{12}W_iW_j''\Phi_n^T\Phi_n \end{array} \end{array}\right)$$

$$S_1=\sum_{i=1}^{r}\sum_{j=1}^{r}Y_{ij},\quad S_2=\sum_{i=1}^{r}\sum_{j=1}^{r}Z_{ij}$$

$$Y_{ij}=\int_{-\frac{b}{2}}^{\frac{b}{2}}\int_{0}^{A}\begin{pmatrix} 0 & 0 & \begin{array}{l} J_{13}U_i'W_j'\Phi_n^T\Phi_n+J_{14}U_iW_j'\dot{\Phi}_n^T\Phi_n \\ +J_{23}U_i'W_j\Phi_n^T\dot{\Phi}_n+J_{24}U_iW_j\dot{\Phi}_n^T\dot{\Phi}_n \end{array} \\ & 0 & \begin{array}{l} J_{15}V_i'W_j'\Phi_n^T\Phi_n+J_{16}V_iW_j'\dot{\Phi}_n^T\Phi_n \\ +J_{25}V_i'W_j\Phi_n^T\dot{\Phi}_n+J_{26}V_iW_j\dot{\Phi}_n^T\dot{\Phi}_n \end{array} \\ & \text{对称} & \begin{array}{l} J_{11}W_i'W_j'\Phi_n^T\Phi_n+J_{12}W_iW_j'\dot{\Phi}_n^T\Phi_n \\ +J_{12}W_i'W_j\Phi_n^T\dot{\Phi}_n+J_{22}W_iW_j\dot{\Phi}_n^T\dot{\Phi}_n \end{array} \end{pmatrix}$$

$$Z_{ij}=\int_{-\frac{b}{2}}^{\frac{b}{2}}\int_{0}^{A}\begin{pmatrix} 0 & 0 & 0 \\ & 0 & 0 \\ & \text{对称} & \begin{array}{l} L_{11}W_i'W_j'\Phi_n^T\Phi_n+L_{12}W_iW_j'\dot{\Phi}_n^T\Phi_n \\ +L_{12}W_i'W_j\Phi_n^T\dot{\Phi}_n+L_{22}W_iW_j\dot{\Phi}_n^T\dot{\Phi}_n \end{array} \end{pmatrix}$$

$$L=\begin{pmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{pmatrix}=\begin{pmatrix} \frac{3}{2}A_{11}\left(\frac{\partial w}{\partial x}\right)^2 & \left(\frac{3}{2}A_{12}+3A_{66}\right)\frac{\partial w}{\partial x}\frac{\partial w}{\partial y} \\ \left(\frac{3}{2}A_{12}+3A_{66}\right)\frac{\partial w}{\partial x}\frac{\partial w}{\partial y} & \frac{3}{2}A_{22}\left(\frac{\partial w}{\partial y}\right)^2 \end{pmatrix}$$

$$J=\begin{bmatrix} J_{11} & & & & & \\ J_{12} & J_{22} & & \text{对称} & & \\ J_{13} & J_{23} & 0 & & & \\ J_{14} & J_{24} & 0 & 0 & & \\ J_{15} & J_{25} & 0 & 0 & 0 & \\ J_{16} & J_{26} & 0 & 0 & 0 & 0 \end{bmatrix}=$$

$$\begin{bmatrix} A_{11}\dfrac{\partial u}{\partial x}+A_{12}\dfrac{\partial v}{\partial y} & & & & & \\ A_{66}\left(\dfrac{\partial u}{\partial y}+\dfrac{\partial v}{\partial x}\right) & A_{22}\dfrac{\partial v}{\partial y}+A_{12}\dfrac{\partial u}{\partial x} & & & \text{对称} & \\ A_{11}\dfrac{\partial w}{\partial x} & A_{12}\dfrac{\partial w}{\partial y} & 0 & & & \\ A_{66}\dfrac{\partial w}{\partial y} & A_{66}\dfrac{\partial w}{\partial x} & 0 & 0 & & \\ A_{66}\dfrac{\partial w}{\partial y} & A_{66}\dfrac{\partial w}{\partial x} & 0 & 0 & 0 & \\ A_{12}\dfrac{\partial w}{\partial x} & A_{22}\dfrac{\partial w}{\partial y} & 0 & 0 & 0 & 0 \end{bmatrix}$$

由式(5)、(6)有:

$$\triangle V_p=-wq_w(x,y)\qquad w=\sum_{i=1}^{r}w_i(x)\,B(y)\,C_n d_{ni}$$

这里　$B(y)=[0\quad 0\quad \varphi_n(y)]$

所以　$$V_p=-\int_{-\frac{b}{2}}^{\frac{b}{2}}\int_0^{l}w\,q_w(x,y)\,\mathrm{d}x\mathrm{d}y=-\sum_{i=1}^{r}d_{ni}^{\mathrm{T}}P_{ni}=-d_n^{\mathrm{T}}P_n \tag{9}$$

其中　$P_n=\{P_{n1}\quad P_{n2}\quad P_{n3}\cdots P_{ni}\cdots P_{nr}\}^{\mathrm{T}}$

$$P_{ni}=\int_{-\frac{b}{2}}^{\frac{b}{2}}\int_0^{l}C_n^{\mathrm{T}}B(y)^{\mathrm{T}}w_i(x)\,q_w(x,y)\,\mathrm{d}x\,\mathrm{d}y$$

总能量(条单元)

$$\Pi=U_P+V_P=\frac{1}{2}d_n^{\mathrm{T}}K_n d_n+\frac{1}{6}d_n^{\mathrm{T}}K_{1n}d_n+\frac{1}{12}d_n^{\mathrm{T}}K_{2n}d_n-d_n^{\mathrm{T}}P_n \tag{10}$$

把所有条叠加得:

$$\overline{\Pi}=\overline{U}_P+\overline{V}_P,\quad \overline{U}_P=\sum_s U_P,\quad \overline{V}_P=\sum_s V_P(13)$$

所以　$$\overline{\Pi}=\frac{1}{2}\overline{d}^{\mathrm{T}}\overline{K}\,\overline{d}+\frac{1}{6}\overline{d}^{\mathrm{T}}\overline{K}_1\overline{d}+\frac{1}{12}\overline{d}^{\mathrm{T}}\overline{K}_2\overline{d}-\overline{d}^{\mathrm{T}}\overline{P}(13) \tag{11}$$

$$\overline{K}=\sum_S K_n,\quad \overline{K}_1=\sum_S K_{1n},\quad \overline{K}_2=\sum_S K_{2n}(13)$$

(11)式引入边界条件,由最小势能原理得

$$\left(\overline{K}+\frac{1}{2}\overline{K}_1+\frac{1}{3}\overline{K}_2\right)\overline{d}-\overline{P}=0 \tag{12}$$

3 Newton-Raphson 迭代法

令 $F(\bar{d}) = (\bar{K} + \frac{1}{2}\bar{K}_1 + \frac{1}{3}\bar{K}_2)\bar{d} - \bar{P} = 0$。 (13)

假设 $\bar{d}_i$ 为方程的解，代入式(13)得 $F(\bar{d}_i) \neq 0$，把 $F(\bar{d}_i)$ 展成泰勒级数得其改善的 $\bar{d}_{i+1}$

即 $$F(\bar{d}_{i+1}) = F(\bar{d}_i) + \frac{\partial F(\bar{d}_i)}{\partial \bar{d}_i}(\bar{d}_{i+1} - \bar{d}_i) = 0。 \quad (14)$$

对式(13)求导得 $\frac{\partial F(\bar{d}_i)}{\partial \bar{d}_i} = \bar{K} + \bar{K}_1 + \bar{K}_2 = \bar{K}_T$。

$\bar{K}_T$ 是在 $\bar{d}_i$ 的切向刚度，代入式(14)有

$$\bar{K}_T \triangle \bar{d}_i = - F_i(\bar{d}_i), \quad (15)$$

$\triangle \bar{d}_i$ 是第 i 次迭代的修正项

所以 $$\bar{d}_{i+1} = \bar{d}_i + \triangle \bar{d}_i。 \quad (16)$$

当 $\frac{1}{N}\sum_{K=1}^{N}\left(\frac{\triangle \bar{d}_K}{\bar{d}_K + \triangle \bar{d}_K}\right)^2 < 0.000\,1$ 时停止迭代。

N 为板的总自由度数目，$\bar{d}_K$ 是 $\bar{d}$ 的第 K 个自由度，$\triangle \bar{d}_K$ 为一次迭代过程中的改变量。

4 算 例

四层正交对称复合材料层合板〔0°/90°/90°/0°〕，受均布荷载，四边简支，边长 $A = b = 30.48$ cm，厚 $h = 0.243$ cm，材料常数 $E_1 = 1.280 \times 10^5$ kg/cm^2，$E_2 = 1.290 \times 10^5$ kg/cm^2，$G_{12} = G_{13} = G_{23} = 2.2 \times 10^4$ kg/cm^2，$u = 0.239\,49$。

边界条件：$x = 0, x = A$ 时，$u = v = w = 0$。位移表达式中级数项部分为 $U_i(x) = V_i(x) = W_i(x) = \sin(i\pi x/A)$，即满足边界条件，计算得出的荷载 - 中心挠度曲线与实验解。见文献〔3〕，经典解析度比较，线性解与非线生解都吻合比较好，如图 2。

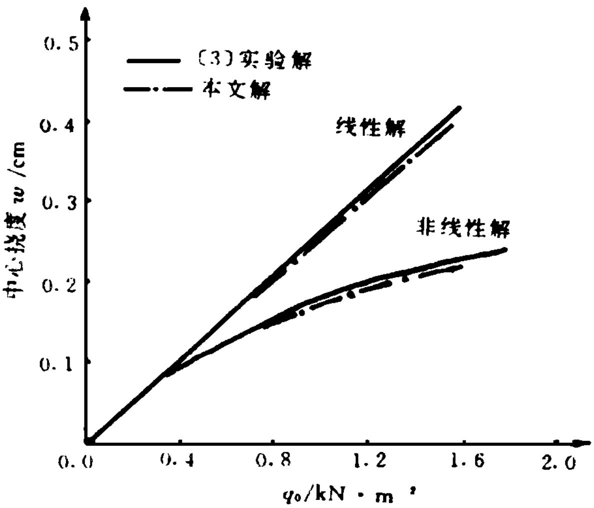

图 2 荷载 - 中心挠度曲线图

5 结束语

在工程结构中，由于复合材料的比强度和比刚度高，层合板常为薄壁轻结构，因此容易出现大变形、小应变为特点的大挠度弯曲，从本文的算例可以得出，当板的中心挠度达到与板厚同量级时，用经典解析解计算的中心挠度将会产生较大的误差。因此，考虑复合材料层合板的几何非线性角是有重要的意义。

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Geomertically Nonlinear Analysis of the Laminated Plates with Fiber Reinforced Composites

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Abstract The geometrically nonlinear problem of laminated plates with fiber reinforced composite is analysised with finite strip method and the nonlinear equations are resolved using the Newton-Raphson method. Through the comparison of the experimenatal results, numerical results are proved satisfactory.

Key words lamnimated plates, geometrically nonlinear, Newton-Raphson iterations.

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Thermal Economical Analysis of Adsorption Refrigeration Systems and the Object Oriented Model

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Abstract Thermal Economical analysis is more reasonable than thermal dynamic analysis for adsorption refrigeration systems when the characteristics and cost of systems are considered comprehensively. In this paper a thermal economical system with heat regenerated adsorption refrigeration system as origin is given according to thermal economical theories. An object oriented model related all sub-systems is presented to speed up the calculation, by which the thermal economical analysis and optimization of adsorption refrigeration systems can be done with fast calculating speed.

Key words adsorption refrigeration; thermal economics; object oriented model