雅可比符号的性质

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摘　要　给出了雅可比符号的新性质:(Ⅰ) $0<a\equiv 0$或$1 \pmod 4$时，雅可比符号$\left(\frac{a}{2ac\pm 1}\right)=1$,雅可比符号$\left(\frac{a}{2ac\pm b}\right)=$雅可比符号$\left(\frac{a}{b}\right)$;(Ⅱ) $0<a\equiv 2$或$3 \pmod 4$时，雅可比符号$\left(\frac{a}{2ac\pm 1}\right)=(-1)^{c}$⒀雅可比符号$\left(\frac{a}{2ac\pm b}\right)=(-1)^{c}\cdot$雅可比符号$\left(\frac{a}{b}\right)$⒀

关键词　雅可比符号;同余方程的解;勒让得符号

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本文中将把雅可比符号$\left(\frac{a}{m}\right)$简单地记为$\left(\frac{a}{m}\right)$⒀众所周知，雅可比符号$\left(\frac{a}{m}\right)$通常有以下7个性质⒀

性质1　当$(a,m)\neq 1$时，$\left(\frac{a}{m}\right)=0$;当$(a,m)=1$时，$\left(\frac{a}{m}\right)=1$或$-1$⒀

性质2　$\left(\frac{1}{m}\right)=1$,$(a,m)=1$时，$\left(\frac{a^2}{m}\right)=1$⒀

性质3　若$a_1\equiv a_2 \pmod m$⒀则$\left(\frac{a_1}{m}\right)=\left(\frac{a_2}{m}\right)$⒀

性质4　若整数$a=a_1a_2\cdots a_n$,则$\left(\frac{a}{m}\right)=\left(\frac{a_1}{m}\right)\left(\frac{a_2}{m}\right)\cdots\left(\frac{a_n}{m}\right)$⒀

性质5　$\left(\frac{-1}{m}\right)=(-1)^{\frac{m-1}{2}}$,$\left(\frac{2}{m}\right)=(-1)^{\frac{1}{8}(m^2-1)}$⒀

性质6　若m和n都是大于1的奇数，则

$$\left(\frac{a}{mn}\right)=\left(\frac{a}{m}\right)\left(\frac{a}{n}\right)⒀$$

性质7　若m和n都是大于1的奇数，则

$$\left(\frac{n}{m}\right)=(-1)^{\frac{n-1}{2}\cdot\frac{m-1}{2}}\left(\frac{m}{n}\right)⒀$$

讨论雅可比符号的性质是为了计算$\left(\frac{a}{m}\right)$的值⒀众所周知，雅可比符号的值与同余方程x^2

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$\equiv a(\bmod m)$ 是否有解关系密切。特别是$(p,a)=1$,p 是奇素数时,$\left(\frac{a}{p}\right)$ 的值是同余方程 $x^2\equiv a(\bmod p)$ 是否有解的必要与充分条件。雅可比符号的作用和重要性就在于此。这里要指出的是,根据以上的7个性质很容易算出 $\left(\frac{a}{m}\right)$ 的值来,而且人们也因此对这种算法很满意。但是这不等于说这种算法就一点缺点也没有,无需加以改进。事实上,在 $\left(\frac{a}{m}\right)$ 中分子 a 是偶数时,按现在流行的算法计算就一定要把 $\left(\frac{a}{m}\right)$ 分成两个雅可比符号来算,如 $\left(\frac{10}{21}\right)=\left(\frac{2}{21}\right)\left(\frac{5}{21}\right)$ 或 $\left(\frac{10}{21}\right)=\left(\frac{-1}{21}\right)\left(\frac{11}{21}\right)$,不这样做就算不出结果来。但缺点是使计算不方便。要克服这个缺点就必须进一步研究,发掘出雅可比符号的新性质来。下面是雅可比符号的新性质。

性质8　若 a,b,c 都是正整数,b 是奇数,$a\equiv 0$或$1(\bmod 4)$,则 $\left(\frac{a}{2ac\pm 1}\right)=1$,$\left(\frac{a}{2ac\pm b}\right)=\left(\frac{a}{b}\right)$。

证明　当 $a\equiv 1(\bmod 4)$ 时,$a=4n+1$。

由性质7、性质3和性质2知

$$\left(\frac{a}{2ac+1}\right)=(-1)^{\frac{a-1}{2}\cdot\frac{2ac+1-1}{2}}\left(\frac{2ac+1}{a}\right)=(-1)^{2nac}\left(\frac{1}{a}\right)=\left(\frac{1}{a}\right)=1。$$

由性质7、性质3和性质5知

$$\left(\frac{a}{2ac-1}\right)=(-1)^{\frac{a-1}{2}\cdot\frac{2ac-1-1}{2}}\left(\frac{2ac-1}{a}\right)=(-1)^{2n(ac-1)}\left(\frac{-1}{a}\right)=\left(\frac{-1}{a}\right)=(-1)^{\frac{a-1}{2}}=1。$$

同理

$$\left(\frac{a}{2ac+b}\right)=(-1)^{\frac{a-1}{2}\cdot\frac{2ac+b-1}{2}}\left(\frac{2ac+b}{a}\right)=(-1)^{n(2ac+b-1)}\left(\frac{b}{a}\right)=$$

$$\left(\frac{b}{a}\right)=(-1)^{\frac{a-1}{2}\cdot\frac{b-1}{2}}\left(\frac{a}{b}\right)=(-1)^{n(b-1)}\left(\frac{a}{b}\right)=\left(\frac{a}{b}\right)。$$

$$\left(\frac{a}{2ac-b}\right)=(-1)^{\frac{a-1}{2}\cdot\frac{2ac-b-1}{2}}\left(\frac{2ac-b}{a}\right)=(-1)^{n(2ac-b-1)}\left(\frac{-b}{a}\right)=\left(\frac{-b}{a}\right)=$$

$$\left(\frac{-1}{a}\right)\left(\frac{b}{a}\right)=(-1)^{\frac{a-1}{2}}\left(\frac{b}{a}\right)=\left(\frac{b}{a}\right)=(-1)^{\frac{a-1}{2}\cdot\frac{b-1}{2}}\left(\frac{a}{b}\right)=\left(\frac{a}{b}\right)。$$

当 $a\equiv 0(\bmod 4)$ 时,$a=2^\alpha u$,$\alpha>1$,u 是奇数。由性质4、性质5和性质7知

$$\left(\frac{a}{2ac+1}\right)=\left(\frac{2^\alpha}{2ac+1}\right)\left(\frac{u}{2ac+1}\right)=(-1)^{\frac{(2ac+1)^2-1}{8}\cdot\alpha}\cdot(-1)^{\frac{u-1}{2}\cdot\frac{2ac+1-1}{2}}\left(\frac{2ac+1}{u}\right)=$$

$$(-1)^{2^{\alpha-1}uc(ac+1)\alpha}\cdot(-1)^{(u-1)2^{\alpha-1}uc}\left(\frac{1}{u}\right)=\left(\frac{1}{u}\right)=1。$$

同理

$$\left(\frac{a}{2ac-1}\right)=\left(\frac{2^\alpha}{2ac-1}\right)\left(\frac{u}{2ac-1}\right)=(-1)^{\frac{(2ac-1)^2-1}{8}\cdot\alpha}\cdot(-1)^{\frac{u-1}{2}\cdot\frac{2ac-1-1}{2}}\left(\frac{2ac-1}{u}\right)=$$

$$(-1)^{2^{\alpha-1}uc(ac-1)\alpha}\cdot(-1)^{\frac{(u-1)(ac-1)}{2}}\left(\frac{-1}{u}\right)=(-1)^{\frac{(u-1)(ac-1)}{2}}\cdot(-1)^{\frac{u-1}{2}}=(-1)^{2^{\alpha-1}uc(u-1)}=1。$$

$$\left(\frac{a}{2ac+b}\right)=\left(\frac{2^\alpha}{2ac+b}\right)\left(\frac{u}{2ac+b}\right)=(-1)^{\frac{(2ac+b)^2-1}{8}\cdot\alpha}\cdot(-1)^{\frac{u-1}{2}\cdot\frac{2ac+b-1}{2}}\left(\frac{2ac+b}{u}\right)=$$

$$(-1)^{2^{\alpha-1}uc(ac+b)\alpha}\cdot(-1)^{\frac{(b^2-1)\alpha}{8}}\cdot(-1)^{\frac{u-1}{2}\cdot\frac{b-1}{2}}\left(\frac{b}{u}\right)=$$

$$(-1)^{\frac{(b^2-1)\alpha}{8}}\cdot(-1)^{\frac{u-1}{2}\cdot\frac{b-1}{2}}\cdot(-1)^{\frac{u-1}{2}\cdot\frac{b-1}{2}}\left(\frac{u}{b}\right)=$$

$$(-1)^{\frac{(u-1)(b-1)}{2}}\left(\frac{2^{\alpha}}{b}\right)\left(\frac{u}{b}\right)=\left(\frac{2^{\alpha}}{b}\right)\left(\frac{u}{b}\right)=\left(\frac{a}{b}\right)$$⒀

$$\left(\frac{a}{2ac-b}\right)=\left(\frac{2^{\alpha}}{2ac-b}\right)\left(\frac{u}{2ac-b}\right)=(-1)^{\frac{(2ac-b)^2-1}{8}\cdot\alpha}\cdot(-1)^{\frac{u-1}{2}\cdot\frac{2ac-b-1}{2}}\left(\frac{2ac-b}{u}\right)=$$

$$(-1)^{2^{\alpha-1}uc(ac-b)\alpha}\cdot(-1)^{\frac{(b^2-1)}{8}\alpha}\cdot(-1)^{\frac{u-1}{2}\cdot\frac{-b-1}{2}}\left(\frac{-b}{u}\right)=$$

$$(-1)^{\frac{(b^2-1)}{8}\alpha}\cdot(-1)^{\frac{u-1}{2}\cdot\frac{-b-1}{2}}\left(\frac{-1}{u}\right)\left(\frac{b}{u}\right)=$$

$$(-1)^{\frac{u-1}{2}\cdot\frac{-b-1}{2}}\cdot(-1)^{\frac{u-1}{2}}\left(\frac{b}{u}\right)\left(\frac{2^{\alpha}}{b}\right)=(-1)^{\frac{u-1}{2}\cdot\frac{1-b}{2}}\cdot(-1)^{\frac{u-1}{2}\cdot\frac{b-1}{2}}\left(\frac{u}{b}\right)\left(\frac{2^{\alpha}}{b}\right)=$$

$$(-1)^{0}\left(\frac{u}{b}\right)\left(\frac{2^{\alpha}}{b}\right)=\left(\frac{a}{b}\right)$$⒀

证毕

性质9 若 a,b,c 都是正整数⒀并且 b 是奇数，$a\equiv 2$或$3(\bmod 4)$，则$\left(\frac{a}{2ac\pm b}\right)=(-1)^{c}\left(\frac{a}{b}\right)$，$\left(\frac{a}{2ac\pm 1}\right)=(-1)^{c}$⒀

证明 当 $a\equiv 2(\bmod 4)$ 时，那末 $a=4n+2$⒀

$$\left(\frac{a}{2ac+1}\right)=\left(\frac{2}{2ac+1}\right)\left(\frac{2n+1}{2ac+1}\right)=(-1)^{\frac{(2ac+1)^2-1}{8}}\cdot(-1)^{nac}\left(\frac{2ac+1}{2n+1}\right)=$$

$$(-1)^{(2n+1)c(ac+1)}\left(\frac{1}{2n+1}\right)=(-1)^{(2n+1)c(ac+1)}=(-1)^{c}$$⒀

$$\left(\frac{a}{2ac-1}\right)=\left(\frac{2}{2ac-1}\right)\left(\frac{2n+1}{2ac-1}\right)=(-1)^{\frac{(2ac-1)^2-1}{8}}$$⒀

$$(-1)^{n(ac-1)}\left(\frac{2ac-1}{2n+1}\right)=(-1)^{(2n+1)c(ac-1)}\cdot(-1)^{n(ac-1)}\left(\frac{-1}{2n+1}\right)=$$

$$(-1)^{c}\cdot(-1)^{n(ac-1)}\cdot(-1)^{n}=(-1)^{c}\cdot(-1)^{nac}=(-1)^{c}$$⒀

$$\left(\frac{a}{2ac+b}\right)=\left(\frac{2}{2ac+b}\right)\left(\frac{2n+1}{2ac+b}\right)=(-1)^{\frac{(2ac+b)^2-1}{8}}\cdot(-1)^{\frac{n(2ac+b-1)}{2}}\left(\frac{2ac+b}{2n+1}\right)=$$

$$(-1)^{(2n+1)c(ac+b)}\cdot(-1)^{\frac{b^2-1}{8}}\cdot(-1)^{\frac{n(b-1)}{2}}\left(\frac{b}{2n+1}\right)=$$

$$(-1)^{c}\left(\frac{2}{b}\right)\cdot(-1)^{\frac{n(b-1)}{2}}\cdot(-1)^{\frac{n(b-1)}{2}}\left(\frac{2n+1}{b}\right)=$$

$$(-1)^{c}\left(\frac{2}{b}\right)\cdot(-1)^{n(b-1)}\left(\frac{2n+1}{b}\right)=(-1)^{c}\left(\frac{2}{b}\right)\left(\frac{2n+1}{b}\right)=(-1)^{c}\left(\frac{a}{b}\right)$$⒀

$$\left(\frac{a}{2ac-b}\right)=\left(\frac{2}{2ac-b}\right)\left(\frac{2n+1}{2ac-b}\right)=(-1)^{\frac{(2ac-b)^2-1}{8}}\cdot(-1)^{\frac{n(2ac-b-1)}{2}}\left(\frac{2ac-b}{2n+1}\right)=$$

$$(-1)^{(2n+1)c(ac-b)}\cdot(-1)^{\frac{b^2-1}{8}}\cdot(-1)^{\frac{n(-b-1)}{2}}\left(\frac{-b}{2n+1}\right)=$$

$$(-1)^{c}\left(\frac{2}{b}\right)\cdot(-1)^{\frac{n(-b-1)}{2}}\left(\frac{-1}{2n+1}\right)\left(\frac{b}{2n+1}\right)=$$

$$(-1)^{c}\left(\frac{2}{b}\right)\cdot(-1)^{\frac{n(-b-1)}{2}}\cdot(-1)^{n}\left(\frac{b}{2n+1}\right)=$$

$$(-1)^{c}\left(\frac{2}{b}\right)\cdot(-1)^{\frac{n(1-b)}{2}}\cdot(-1)^{\frac{n(b-1)}{2}}\left(\frac{2n+1}{b}\right)=$$

$$(-1)^c \cdot (-1) \circ \left(\frac{2}{b}\right)\left(\frac{2n+1}{b}\right) = (-1)^c \left(\frac{a}{b}\right).$$

当 $a \equiv 3 (\bmod 4)$ 时，那末 $a = 4n+3$⒀

$$\left(\frac{a}{2ac+1}\right) = (-1)^{\frac{a-1}{2}\cdot\frac{2ac+1-1}{2}}\left(\frac{2ac+1}{a}\right) = (-1)^{(2n+1)ac}\left(\frac{1}{a}\right) = (-1)^c\left(\frac{1}{a}\right) = (-1)^c ⒀$$

$$\left(\frac{a}{2ac-1}\right) = (-1)^{\frac{a-1}{2}\cdot\frac{2ac-1-1}{2}}\left(\frac{2ac-1}{a}\right) = (-1)^{(2n+1)(ac-1)}\left(\frac{-1}{a}\right) =$$

$$(-1)^{(2n+1)(ac-1)} \cdot (-1)^{\frac{a-1}{2}} = (-1)^{(2n+1)(ac-1)} \cdot (-1)^{2n+1} = (-1)^{(2n+1)ac} = (-1)^c ⒀$$

$$\left(\frac{a}{2ac+b}\right) = (-1)^{\frac{a-1}{2}\cdot\frac{2ac+b-1}{2}}\left(\frac{2ac+b}{a}\right) = (-1)^{\frac{(2n+1)(2ac+b-1)}{2}}\left(\frac{b}{a}\right) =$$

$$(-1)^{\frac{(2n+1)(2ac+b-1)}{2}} \cdot (-1)^{\frac{(2n+1)(b-1)}{2}}\left(\frac{a}{b}\right) =$$

$$(-1)^{(2n+1)(ac+b-1)}\left(\frac{a}{b}\right) = (-1)^{(2n+1)ac}\left(\frac{a}{b}\right) = (-1)^c\left(\frac{a}{b}\right) ⒀$$

$$\left(\frac{a}{2ac-b}\right) = (-1)^{\frac{a-1}{2}\cdot\frac{2ac-b-1}{2}}\left(\frac{2ac-b}{a}\right) = (-1)^{\frac{(2n+1)(2ac-b-1)}{2}}\left(\frac{-b}{a}\right) =$$

$$(-1)^{\frac{(2n+1)(2ac-b-1)}{2}}\left(\frac{-1}{a}\right)\left(\frac{b}{a}\right) = (-1)^{\frac{(2n+1)(2ac-b-1)}{2}} \cdot (-1)^{\frac{a-1}{2}} \cdot (-1)^{\frac{a-1}{2}\cdot\frac{b-1}{2}}\left(\frac{a}{b}\right) =$$

$$(-1)^{\frac{(2n+1)(2ac-b-1)}{2}} \cdot (-1)^{(2n+1)\cdot\frac{b+1}{2}}\left(\frac{a}{b}\right) = (-1)^{(2n+1)ac}\left(\frac{a}{b}\right) = (-1)^c\left(\frac{a}{b}\right) ⒀$$

证毕

性质10　若 a, b, c 都是正整数，b 是奇数，则 $\left(\frac{a}{4ac\pm1}\right) = 1$，$\left(\frac{a}{4ac\pm b}\right) = \left(\frac{a}{b}\right)$⒀

从性质8和性质9看来，性质10显然成立⒀

假设雅可比符号 $\left(\frac{a}{m}\right)$ 中 a 是正整数，由性质8和性质9知

(Ⅰ) $m = 2ac \pm b$⒀$< b \leqslant a$ 时

$$\left(\frac{a}{m}\right) = \left(\frac{a}{2ac\pm b}\right) = \begin{cases} \left(\frac{a}{b}\right), & a \equiv 0 或 1 (\bmod 4) \\ (-1)^c\left(\frac{a}{b}\right) ⒀ & a \equiv 2 或 3 (\bmod 4) \end{cases} \tag{1}$$

(Ⅱ) $m = 2ac \pm 1$时

$$\left(\frac{a}{m}\right) = \left(\frac{a}{2ac\pm1}\right) = \begin{cases} 1, & a \equiv 0 或 1 (\bmod 4) \\ (-1)^c ⒀ & a \equiv 2 或 3 (\bmod 4) \end{cases} \tag{2}$$

按照式(1)和(2)可以得到计算雅可比符号 $\left(\frac{a}{m}\right)$ 的一个新方法，本文中把现在流行的计算雅可比符号 $\left(\frac{a}{m}\right)$ 的方法称为旧方法⒀

例1　用新旧两种方法计算 $\left(\frac{46}{91}\right)$⒀

解　(1) 用旧方法计算

$$\left(\frac{46}{91}\right) = \left(\frac{2}{91}\right)\left(\frac{23}{91}\right) = (-1)^{\frac{91^2-1}{8}} \cdot (-1)^{\frac{23-1}{2}\cdot\frac{91-1}{2}}\left(\frac{91}{23}\right) = \left(\frac{-1}{23}\right) = (-1)^{\frac{23-1}{2}} = -1 ⒀$$

(2) 用新方法计算

$$\left(\frac{46}{91}\right) = \left(\frac{46}{92\cdot1-1}\right) \xlongequal{46=4n+2} (-1)^1 = -1 ⒀$$

例2 用新旧两种方法计算$\left(\frac{163}{257}\right)$⒀

解 (1) 用旧方法计算⒀

$$\left(\frac{163}{257}\right)=(-1)^{\frac{163-1}{2}\cdot\frac{257-1}{2}}\left(\frac{257}{163}\right)=\left(\frac{94}{163}\right)=\left(\frac{2}{163}\right)\left(\frac{47}{163}\right)=$$

$$(-1)^{\frac{163^2-1}{8}}(-1)^{\frac{47-1}{2}\cdot\frac{163-1}{2}}\left(\frac{163}{47}\right)=$$

$$\left(\frac{22}{47}\right)=\left(\frac{2}{47}\right)\left(\frac{11}{47}\right)=(-1)^{\frac{47^2-1}{8}}\cdot(-1)^{\frac{47-1}{2}\cdot\frac{11-1}{2}}\left(\frac{47}{11}\right)=$$

$$-\left(\frac{3}{11}\right)=-(-1)^{\frac{3-1}{2}\cdot\frac{11-1}{2}}\left(\frac{11}{3}\right)=\left(\frac{2}{3}\right)=(-1)^{\frac{3^2-1}{8}}=-1$$⒀

(2) 用新方法计算

$$\left(\frac{163}{257}\right)=\left(\frac{163}{326\cdot1-69}\right)\xlongequal{163=4n+3}(-1)^{1}\left(\frac{163}{69}\right)=-\left(\frac{25}{69}\right)=-1$$⒀

例3 用新旧两种方法计算$\left(\frac{5608}{7649}\right)$⒀

解 (1) 用旧方法计算

$$\left(\frac{5608}{7649}\right)\xlongequal{5608=4n}\left(\frac{1402}{7649}\right)=\left(\frac{2}{7649}\right)\left(\frac{701}{7649}\right)=(-1)^{\frac{7649^2-1}{8}}\left(\frac{701}{7649}\right)=$$

$$(-1)^{\frac{701-1}{2}\cdot\frac{7649-1}{2}}\left(\frac{7649}{701}\right)=\left(\frac{639}{701}\right)=(-1)^{\frac{639-1}{2}\cdot\frac{701-1}{2}}\left(\frac{701}{639}\right)=\left(\frac{62}{639}\right)=$$

$$\left(\frac{2}{639}\right)\left(\frac{31}{639}\right)=(-1)^{\frac{639^2-1}{8}}\cdot(-1)^{\frac{31-1}{2}\cdot\frac{639-1}{2}}\left(\frac{639}{31}\right)=-\left(\frac{19}{31}\right)=-(-1)^{\frac{19-1}{2}\cdot\frac{31-1}{2}}\left(\frac{31}{19}\right)=$$

$$\left(\frac{12}{19}\right)=\left(\frac{3}{19}\right)=(-1)^{\frac{3-1}{2}\cdot\frac{19-1}{2}}\left(\frac{19}{3}\right)=-\left(\frac{1}{3}\right)=-1$$⒀

(2) 用新方法计算

$$\left(\frac{5608}{7649}\right)\xlongequal{5608=4n}\left(\frac{1402}{7649}\right)=\left(\frac{1402}{2804\cdot3-763}\right)\xlongequal{1402=4n+2}-\left(\frac{1402}{763}\right)=$$

$$-\left(\frac{639}{763}\right)=-\left(\frac{639}{1278\cdot1-515}\right)\xlongequal{639=4n+3}\left(\frac{639}{515}\right)=\left(\frac{124}{515}\right)\xlongequal{124=4n}\left(\frac{31}{515}\right)=$$

$$\left(\frac{31}{62\cdot8+19}\right)\xlongequal{31=4n+3}\left(\frac{31}{19}\right)=\left(\frac{12}{19}\right)\xlongequal{12=4n}\left(\frac{3}{19}\right)=\left(\frac{3}{6\cdot3+1}\right)\xlongequal{3=4n+3}(-1)^{3}=-1$$⒀

例4 用新旧两种方法计算$\left(\frac{365}{1847}\right)$⒀

解 (1) 用旧方法计算

$$\left(\frac{365}{1847}\right)=(-1)^{\frac{365-1}{2}\cdot\frac{1847-1}{2}}\left(\frac{1847}{365}\right)=\left(\frac{22}{365}\right)=\left(\frac{2}{365}\right)\left(\frac{11}{365}\right)=$$

$$(-1)^{\frac{365^2-1}{8}}\cdot(-1)^{\frac{11-1}{2}\cdot\frac{365-1}{2}}\left(\frac{365}{11}\right)=-\left(\frac{2}{11}\right)=-(-1)^{\frac{11^2-1}{8}}=1$$⒀

(2) 用新方法计算

$$\left(\frac{365}{1847}\right)=\left(\frac{365}{730\cdot3-343}\right)\xlongequal{365=4n+1}\left(\frac{365}{343}\right)=\left(\frac{22}{343}\right)=$$

$$\left(\frac{22}{44\cdot8-9}\right)\xlongequal{22=4n+2}(-1)^{8}\left(\frac{22}{9}\right)=\left(\frac{4}{9}\right)=1$$⒀

从上面的两种计算方法的对比中可以看出,用新方法计算雅可比符号比用旧方法计算好,因为,计算过程简化了一点⒀主要的优点是$\left(\frac{a}{m}\right)$中分子$a>0$是偶数时,用新方法计算可以不把$\left(\frac{a}{m}\right)$分为两个计算也可以算出结果来,而且不要用性质7颠倒雅可比符号中的分子和分母进行计算⒀

就计算而言,雅可比符号优于勒让得符号之处是不需要考虑其中的分子是不是奇素数⒀现在雅可比符号有了性质8和性质9,这说明雅可比符号还有一个优于勒让得符号之处是不需要考虑其分子是奇数还是偶数⒀

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The Summary for the Properties of the Jacobi′s Symbol

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Abstract This article presents the new properties of the Jacobi′s symbol:(1) When $0<a\equiv 0$ or $1 \pmod 4$, the Jacobi′s symbol $\left(\frac{a}{2ac\pm 1}\right)=1$, Jacobi′s symbol $\left(\frac{a}{2ac\pm b)}\right)=$ Jacobi′s symbol $\left(\frac{a}{b}\right)$; (2) When $0<a\equiv 2$ or $3 \pmod 4$, the Jacobi′s symbol $\left(\frac{a}{2ac\pm 1}\right)=(-1)^{c}$, Jacobi′s symbol $\left(\frac{a}{2ac\pm b}\right)=(-1)^{c}\cdot$ Jacobi′s symbol $\left(\frac{a}{b}\right)$.

Key Words Jacobi′s symbol;solution of congruent eguation;legendre′s symbol.