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基于完全剩余格值逻辑上的不分明化环

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摘要: 运用应明生教授提出的完全剩余格值逻辑 L 语义的方法引入了不分明化环的概念,并且讨论了这类环(L-环)的一些性质。

关 键 词: L-逻辑; 不分明化环; 连续值逻辑

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0 引 言

在文献[2]中,我们提出了基于连续值逻辑上的不分明化环的概念,并得到了它的一些性质。作为连续值逻辑的更为一般情形——格值逻辑 L,本文运用完全剩余格值逻辑 L 语义的方法进一步引入了 L-不分明化环,同时讨论它的一些代数性质。

文中的逻辑术语、符号和逻辑公式均源于文献[1]。

1 L-不分明化环

定义 1.1 设 R 是一个分明环,称一元 F-谓词 $FR \in \mathcal{F}_L(\mathcal{F}_L(R))$ 为 R 的不分明子环(简称为 R 的不分明环),其定义如下:

$$FR(A) := (\forall x, y \in R)((x \in A) < (y \in A) \to (x + y \in A) < (-x \in A) < (xy \in A))$$

定理 1.2 设 R 是一分明环,则对任意的 $A \in \mathcal{F}_L(R)$,有

$$\models FR(A) \leftrightarrow (A + A \subseteq A) < (A \subseteq -A) < (AA \subseteq A)$$

证明

$$[A + A \subseteq A] = \prod_{z \in R}((A + A)(z) \propto A(z)) = \prod_{z \in R}(\prod_{\substack{x, y \in R \\ x + y = z}} A(x)A(y) \propto A(z))$$

$$= \prod_{z \in R}(\sum_{\substack{x, y \in R \\ x + y = z}}(A(x)A(y) \propto A(z)) = \prod_{x, y \in R}(A(x)A(y) \propto A(x + y))$$

$$[A \subseteq -A] = \prod_{z \in R}(A(z) \propto (-A)(z)) = \prod_{z \in R}(A(z) \propto \sum_{\substack{x \in R \\ -x = z}} A(x)) = \prod_{z \in R}(A(z) \propto A(-z))$$

$$[AA \subseteq -A] = \prod_{z \in R}(AA(z) \propto A(z)) = \prod_{z \in R}(\sum_{\substack{x, y \in R \\ x + y = z}} A(x)A(y) \propto A(z))$$

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$$= \prod_{z\in R}\prod_{\substack{x,y\in R\\ xy=z}}(A(x)A(y)\propto A(z)) = \prod_{x,y\in R}(A(x)A(y)\propto A(xy))$$

$$[(A+A\subseteq A)<(AA\subseteq A)] = \prod_{x,y\in R}(A(x)A(y)\propto A(x+y))\prod_{x,y\in R}(A(x)A(y)\propto A(xy))$$

$$= \prod_{x,y\in R}((A(x)A(y)\propto A(x+y))(A(x)A(y)\propto A(xy))$$

$$= \prod_{x,y\in R}(A(x)A(y)\propto A(x+y)A(xy))$$

$$[(A+\subseteq -A] = \prod_{z\in R}(A(z)\propto A(-z)) \geqslant \prod_{x,y\in R}(A(x)A(y)\propto A(-x))$$

一方面

$$[(A\subseteq -A)<(A+A\subseteq A)<(AA\subseteq A)]$$

$$= \prod_{z\in R}(A(z)\propto A(-z))\prod_{x,y\in R}(A(x)A(y)\propto A(x+y)A(xy))$$

$$= \prod_{x,y\in R}((A(x)\propto A(-x))(A(x)A(y)\propto A(x+y)A(xy)))$$

$$\leqslant \prod_{x,y\in R}(A(x)A(y)\propto A(-x)A(x+y)A(xy))$$

另一方面

$$[(A\subseteq -A)<(A+A\subseteq A)<(AA\subseteq A)]$$

$$= \prod_{z\in R}(A(z)\propto A(-z))\prod_{x,y\in R}(A(x)A(y)\propto A(x+y)A(xy))$$

$$\geqslant \prod_{x,y\in R}(A(x)A(y)\propto A(-x))\prod_{x,y}(A(x)A(y)\propto A(x+y)A(xy))$$

$$= \prod_{x,y\in R}(A(x)A(y)\propto A(-x)A(x+y)A(xy))$$

于是,

$$[(A\subseteq -A)<(A+A\subseteq A)<(AA\subseteq A)]$$

$$= \prod_{x,y\in R}(A(x)A(y)\propto A(-x)A(x+y)A(xy)) = [FR(A)]$$

定理 1.3　设 R 是一分明环,则对 R 的任意一族 L-F 子集 $\{A_i\}_{i\in S}$,有 $\models(\forall i)(i\in S\to FR(A_i))\to FR(\bigcap_{i\in S}A_i)$

证明　$[FR(\bigcap_{i\in S}A_i)] = \prod_{x,y\in R}(\bigcap_{i\in S}A_i(x)\bigcap_{i\in S}A_i(y)\propto\bigcap_{i\in S}A_i(-x)\bigcap_{i\in S}A_i(x+y)\bigcap_{i\in S}A_i(xy))$

$$= \prod_{x,y\in R}(\prod_{i\in S}A_i(x)\prod_{i\in S}A_i(y)\propto\prod_{i\in S}A_i(-x)\prod_{i\in S}A_i(x+y)\prod_{i\in S}A_i(xy))$$

$$\geqslant \prod_{x,y\in R}(\prod_{i\in S}(A_i(x)A_i(y)\propto A_i(-x)A_i(x+y)A_i(xy))) = [(\forall i)(i\in S\to FR(A_i))]$$

定理 1.4　设 R、T 是两个分明环,$f:R\to T$ 是满同态,则对任意的 $A\in\mathcal{F}_L(R)$,$B\in\mathcal{F}_L(T)$,有

1) $\models FR(A)\to FR(f(A))$; 2) $\models FR(B)\leftrightarrow FR(f^{-1}(B))$[13]

证 明　1) 设 $[FR(f(A))] = \prod_{x,y\in T}(f(A)(x)f(A)(y)\propto f(A)(-x)f(A)(x+y)f(A)(xy))$

$$= \prod_{x,y\in T}\left(\sum_{\substack{z\in R\\ f(z)=x}}A(z)\sum_{\substack{u\in R\\ f(u)=y}}A(u)\propto\sum_{\substack{v\in R\\ f(v)=-x}}A(v)\sum_{\substack{w\in R\\ f(w)=x+y}}A(w)\sum_{\substack{t\in R\\ f(t)=xy}}A(t)\right)$$

$$\geqslant \prod_{x,y\in T}\left(\sum_{\substack{z\in R\\ f(z)=x}} A(z) \sum_{\substack{u\in R\\ f(u)=y}} A(u) \propto \sum_{\substack{z\in R\\ f(z)=x}} A(-z) \sum_{\substack{z,u\in R\\ f(z)=x\\ f(u)=y}} A(z+u) \sum_{\substack{z,u\in R\\ f(z)=x\\ f(u)=y}} A(zu)\right)$$

$$= \prod_{x,y\in T}\left(\sum_{\substack{z,u\in R\\ f(z)=x\\ f(u)=y}} A(z)A(u) \propto \sum_{\substack{z,u\in R\\ f(z)=x\\ f(u)=y}} A(-z)A(z+u)A((zu)\right)$$

$$\geqslant \prod_{x,y\in T}\left(\prod_{zu\in R}(A(z)A(u) \propto A(-z)A(z+u)A(zu)\right)$$

$$= \prod_{z,u\in R}(A(z)A(u) \propto A(-z)A(z+u)A(zu)) = [FR(A)]$$

2) $[FR(f^{-1}(B))]$

$$= \prod_{x,y\in R}(f^{-1}(B)(x)f^{-1}(B)(y) \propto f^{-1}(B)(-x)f^{-1}(B)(x+y)f^{-1}(B)(xy))$$

$$= \prod_{x,y\in R}(B(f(x))B(f(y)) \propto B(f(-x))B(f(x+y))B(f(xy)))$$

$$= \prod_{x,y\in R}(B(f(x))B(f(y)) \propto B(-f(x))B(f(x)+f(y))B(f(x)f(y)))$$

$$= \prod_{u,v\in T}(B(u)B(v) \propto B(-u)B(u+v)B(uv)) = [FR(B)]$$

[参 考 文 献]

[1] *Mingsheng Ying. Fuzzifying topology based on complete residuated lattice-valued logic*(Ⅰ)[*J*]. *F.S.S*, 1993, (56): 337～373.

[2] 蒋志勇⒀不分明化环[*J*]⒀(*accepetd*)⒀

Fuzzifying Ring Based on Complete Residuated Lattice-valued Logic

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Abstract: In this paper, we use the semantic method of complete residuated lattice-valued logic L, proposed by professor Mingsheng Ying, to introduce the concept of fuzzifying ring, and discuss some properties of this kind of ring(L-ring).

Key words: L-logic; fuzzifying ring; continuous-ualued logic