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皇冠图 $G_{n,m}$的邻点可区别边色数

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摘要:定义皇冠图 $G_{n,m}$为$V(G_{n,m})=\{u_i|i=1,2,\cdots,n\}\cup\{v_i|i=1,2,\cdots,n\}\bigcup\limits_{i=1}^{n}\{u_{ij}|j=1,2,\cdots,m\}$,$E(G_{n,m})=\{u_1u_2,u_2u_3,\cdots,u_nu_1\}\cup\{v_1v_2,v_2v_3,\cdots,v_nv_1\}\cup\{u_iv_i|i=1,2,\cdots,n\}\bigcup\limits_{i=1}^{n}\{u_iu_{ij}|j=1,2,\cdots,m\}\bigcup\limits_{i=1}^{n}\{u_{ij}u_{i(j+1)}|j=1,2,\cdots,m-1\})$,$(n\geq3,m\geq1)$.本文得到了 $G_{n,m}$的邻点可区别边色数.

关 键 词:图;皇冠图;邻点可区别边色数
中图分类号:O157.5 **文献标识码**:A

1 引 言

由计算机科学等引出的点可区别染色问题[1-3],是一个十分困难的问题,十多年来在刊物上发表的文章较少.[4]减弱了要求,提出了邻点可区别的边染色问题.本文给出了 $G_{n,m}$的邻点可区别边色数.

2 相关定义及引理

定义 1 [1-4]对图 G 的一个 k-边正常染色法[5]f,若满足$\forall$ u,v$\in V(G)$且 $uv\in E(G)$,$u\neq v$,有 $C(u)\neq C(v)$,则称 f 为 G 的一个 k-邻点可区别的染色法,简记作 k-AVDEC of G,而

$$\chi'_{as}(G)=\min\{k|k-\text{AVDEC of G}\}$$

称为 G 的邻点可区别边色数,其中

$$C(u)=\{f(uw)|uw\in E(G)\}$$

显然有:

$$\chi'_{as}(G)\geq\triangle(G)$$

其中,$\triangle$(G)表示 G 的最大度数.

定义 2 皇冠图 $G_{n,m}$满足条件:

$V(G_{n,m})=\{u_i|i=1,2,\cdots,n\}\cup\{v_i|i=1,2,\cdots,n\}\bigcup\limits_{i=1}^{n}\{u_{ij}|j=1,2,\cdots,m\}$.

$E(G_{n,m})=\{u_1u_2,u_2u_3,\cdots,u_nu_1\}\cup\{v_1v_2,v_2v_3,\cdots,v_nv_1\}\cup\{u_iv_i|i=1,2,\cdots,n\}$

$\bigcup\limits_{i=1}^{n}\{u_iu_{ij}|j=1,2,\cdots,m\}\bigcup\limits_{i=1}^{n}\{u_{ij}u_{i(j+1)}|j=1,2,\cdots,m-1\}$,$(n\geq3,m\geq1)$.

例如图一表示了 $G_{3,3}$

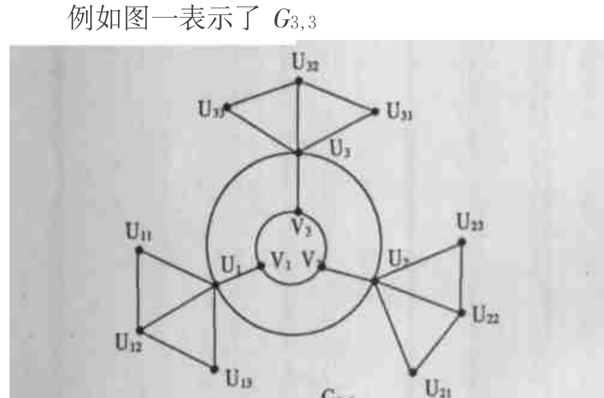

图 1

引理 1[6] 对$|V(G)|\geq3$的连通图 G,若有 $uv\in E(G)$,且 $d(u)=d(v)=\triangle$(G),则有

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$$\chi'_{as}(G)\geq\triangle(G)+1$$

引理2 对皇冠图 $G_{n,m}(n\geq3, m\geq1)$，有

$$\triangle(G_{n,m})=m+3$$

证明 显然 $G_{n,m}$ 中，$u_i\in V_\triangle$，$i=1,2,\cdots,n$，而 $d(u_i)=m+3$，所以 $\triangle(G_{n,m})=m+3$.

文中未加述及的术语、记号可参见[5][6][7].

3 主要结果

定理1 对皇冠图 $G_{n,m}(n\geq3, m\geq1)$，有

$$\chi'_{as}(G_{n,m})=m+4$$

证明 由引理1、引理2知 $\chi'_{as}\geq=m+4$，为了证明定理为真，仅需证明 $G_{n,m}$ 存在 $(m+4)-$AVDEC 法. 分以下两种情况.

情形1 当 $n\equiv1(\mathrm{mod}\ 2)$ 时令 f 为：

$$f(u_iu_{i+1})=f(v_iv_{i+1})=\begin{cases}1,\ i\equiv1(\mathrm{mod}\ 2)\\2,\ i\equiv0(\mathrm{mod}\ 2)\end{cases}i=1,2,\cdots,n-1;$$

$$f(u_nu_1)=f(v_nv_1)=3;$$

$$f(u_iv_i)=\begin{cases}4,\ i\equiv1(\mathrm{mod}\ 2)\\3,\ i\equiv0(\mathrm{mod}\ 2)\end{cases},i=1,2,\cdots,n;$$

$$f(u_iu_{ij})=j+4,i=1,2,\cdots,n,j=1,2,\cdots,m;$$

$$f(u_{ij}u_{i(j+1)})=\begin{cases}1,\ i\equiv1(\mathrm{mod}\ 2)\\2,\ i\equiv0(\mathrm{mod}\ 2)\end{cases},$$

$i=1,2,\cdots,n,j=1,2,\cdots,m-1(m\geq2)$.

对此 f，有

$$C(v_1)=\{1,3,4\};$$

$$C(v_n)=\{2,3,4\};$$

$$C(v_i)=\begin{cases}\{1,2,3\},\ i\equiv0(\mathrm{mod}\ 2)\\\{1,2,4\},\ i\equiv1(\mathrm{mod}\ 2)\end{cases},$$

$i=2,3,\cdots,n-1$;

$$C(u_1)=\{1,3,4\}\cup\{5,6,\cdots,m+4\};$$

$$C(u_n)=\{2,3,4\}\cup\{5,6,\cdots,m+4\};$$

$$C(u_i)=\begin{cases}\{1,2,3\}\cup\{5,6,\cdots,m+4\},\ i\equiv0(\mathrm{mod}\ 2)\\\{1,2,4\}\cup\{5,6,\cdots,m+4\},\ i\equiv1(\mathrm{mod}\ 2)\end{cases},$$

$i=2,3,\cdots,n-1$;

$$C(u_{il})=\begin{matrix}\{5\},m=1\\\{1,5\},m\geq2\end{matrix},i=1,2,\cdots,n;$$

$$C(u_{ij})=\{1,2,j+4\},i=1,2,\cdots,n,j=2,3,\cdots,m-1(m\geq3);$$

$$C(u_{im})=\begin{cases}\{1,m+4\},m\equiv0(\mathrm{mod}\ 2)\\\{2,m+4\},m\equiv1(\mathrm{mod}\ 2)\end{cases},$$

$i=1,2,\cdots,n(m\geq2$

所以 f 是 $G_{n,m}$ 的 $(m+4)-$AVDEC 法，从而此时定理为真.

情形2 当 $n\equiv0(\mathrm{mod}\ 2)$ 时

令 f 为：

$$f(u_iu_{i+1})=f(v_iv_{i+1})=\begin{cases}1,\ i\equiv1(\mathrm{mod}\ 2)\\2,\ i\equiv0(\mathrm{mod}\ 2)\end{cases},$$

$i=1,2,\cdots,n$;

$$f(u_iv_i)=\begin{cases}4,\ i\equiv1(\mathrm{mod}\ 2)\\3,\ i\equiv0(\mathrm{mod}\ 2)\end{cases},$$

$i=1,2,\cdots,n$;

$$f(u_iu_{ij})=j+4,i=1,2,\cdots,n,j=1,2,\cdots,m;$$

$$f(u_{ij}u_{i(j+1)})=\begin{cases}4,j\equiv1(\mathrm{mod}\ 2)\\3,j\equiv0(\mathrm{mod}\ 2)\end{cases},$$

$i=1,2,\cdots,n,j=1,2,\cdots,m-1(m\geq2)$.

对此 f，有

$$C(v_i)=\begin{cases}\{1,2,4\},\ i\equiv1(\mathrm{mod}\ 2)\\\{1,2,3\},\ i\equiv0(\mathrm{mod}\ 2)\end{cases},i=1,2,\cdots,n;$$

$$C(u_il)=\begin{cases}\{5\},m=1\\\{1,5\},m\geq2\end{cases},i=1,2,\cdots,n;$$

$$C(u_{ij})=\{1,2,j+4\},i=1,2,\cdots,n,j=2,3,\cdots,m-1(m\geq3);$$

$$C(u_{im})=\begin{cases}\{1,m+4\},m\equiv0(\mathrm{mod}2)\\\{2,m+4\},m\equiv1(\mathrm{mod}2)\end{cases},i=1,2,\cdots,n(m\geq2.$$

所以 f 是 $G_{n,m}$ 的 $(m+4)-$AVDEC 法，从而此时定理为真.

综上所述，定理成立.

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Adjacent Vertex-Distinguishing Edges Coloring of Crown Graph $G_{n,m}$

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Abstract: Define Crown graph $G_{n,m}$ as $V(G_{n,m})=\{u_i \mid i=1,2,\cdots,n\} \cup \{v_i \mid i=1,2,\cdots,n\} \bigcup_{i=1}^{n} \{u_{ij} \mid j=1,2,\cdots,m\}$, $E(G_{n,m})=\{u_1u_2, u_2u_3, \cdots, u_nu_1\} \cup \{v_1v_2, v_2v_3, \cdots, v_nv_1\} \cup \{u_iv_i \mid i=1,2,\cdots,n\} \bigcup_{i=1}^{n} \{u_iu_{ij} \mid j=1,2,\cdots,m\} \bigcup_{i=1}^{n} \{u_{ij}u_{i(j+1)} \mid j=1,2,\cdots,m-1\}$, $(n \geq 3, m \geq 1)$.

Key words: Graph; Crown graph; Adjacent Vertex-distinguishing Edge Coloring

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(上接第140页)

The Clique Covering Numbers of the k-regular Graph on 10 Vertices Including K_5

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Abstract: It is proved that the maximum clique is K_5 in k-regular graph on 10 vertices, and the clique covering numbers including K_5 have been discussed, based on the interchangeability of vertices.

Key words: clique covering numbers; adjacent; regular; interchangeable.