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L－不分明化环

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摘要:运用应明生教授提出的完全剩余格 L 值逻辑语义的方法引入了 L－不分明化环的概念,然后讨论了 L－环的若干性质.

关　键　词:完全剩余格值逻辑;L－不分明化环

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0　引言

在文献[2]中,我们给出了基于连续值逻辑 L罩上的不分明化环的定义,并得到了它的一些性质.作为连续值逻辑 L罩的更为一般情形一格 L 值逻辑,本文运用完全剩余格 L 值逻辑的语义的方法进一步引入了 L－不分明化环,并得到了它的若干代数性质.

1　L－不分明化环

定义 1.1　设 R 为一个分明环,称一元 F－谓词 $FR\in$云$_L$(云$_L(R)$)为 R 的不分明子环(简称为 F 的不分明环),其定义如下:

$$FR(A)=(\forall x、y\in R)((x\in A)\wedge(y\in A)\rightarrow(x+y\in A)\wedge(-x\in A)\wedge(xy\in A))$$

定理 1.2　设 R 为一个分明环,则对任意的 $A\in$云$_L(R)$,有:

$$\models FR(A)\leftrightarrow(A+A\subseteq A)\wedge(A\subseteq -A)\wedge(AA\subseteq A)$$

证明　$[A+A\subseteq A]=\prod_{z\in R}((A+A)(z)\alpha A(z))=\prod_{z\in R}((\sum_{x+y=z}A(x)\cdot A(y))\alpha A(z))$

$=\prod_{z\in R}\prod_{x+y=z}(A(x)\cdot A(y)\alpha A(z))=\prod_{x、y\in R}(A(x)\cdot A(y)\alpha A(x+y))$

$[AA\subseteq A]=\prod_{z\in R}((AA)(z)\alpha A(z))=\prod_{z\in R}((\sum_{xy=z}A(x)\cdot A(y))\alpha A(z))=\prod_{z\in R}\prod_{xy=z}(A(x)\cdot A(y)\alpha A(z))$

$=\prod_{x、y\in R}(A(x)\cdot A(y)\alpha A(xy))$

$[A\subseteq -A]=\prod_{z\in R}(A(z)\alpha(-A)(z))=\prod_{z\in R}(A(z)\alpha\sum_{-x=z}A(x))=\prod_{z\in R}(A(z)\alpha A(-z))$

$[(A+A\subseteq A)\wedge(AA\subseteq A)]=\prod_{x、y\in R}(A(x)\cdot A(y)\alpha A(x+y))\cdot\prod_{x、y\in R}(A(x)\cdot A(y)\alpha A(xy))$

$=\prod_{x、y\in R}((A(x)\cdot A(y)\alpha A(x+y))\cdot(A(x)\cdot A(y)\alpha A(xy)))=\prod_{x、y\in R}(A(x)\cdot A(y)\alpha A(x+y)\cdot A(xy))$

$[A\subseteq -A]=\prod_{z\in R}(A(z)\alpha A(-z))\geqslant\prod_{x、y\in R}(A(x)\cdot A(y)\alpha A(-x))$

一方面,$[(A\subseteq -A)\wedge(A+A\subseteq A)\wedge(AA\subseteq A)]$

$\geqslant\prod_{x、y\in R}(A(x)\cdot A(y)\alpha A(-x))\cdot\prod_{x、y\in R}(A(x)\cdot A(y)\alpha A(x+y)\cdot A(xy))$

$=\prod_{x、y\in R}(A(x)\cdot A(y)\alpha A(-x)\cdot A(x+y)\cdot A(xy))$

另一方面,$[(A\subseteq -A)\wedge(A+A\subseteq A)\wedge(AA\subseteq A)]$

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$=\prod_{z\in R}(A(z))\alpha A(-z))\bullet\prod_{x、y\in R}(A(x)\bullet A(y)\alpha A(x+y)\bullet A(xy))$
$=\prod_{x、y\in R}((A(x)\alpha A(-x))\bullet(A(x)\bullet A(y)\alpha A(x+y)\bullet A(xy)))$
$\leqslant\prod_{x、y\in R}(A(x)\bullet A(y)\alpha A(-x)\bullet A(x+y)\bullet A(xy))$

于是，

$[(A\subseteq -A)\wedge(A+A\subseteq A)\wedge(AA\subseteq A)]=\prod_{x、y\in R}(A(x)\bullet A(y)\alpha A(-x)\bullet A(x+y)\bullet A(xy))=[\mathrm{FR}(A)]$

定理1.3 设 R 为一个分明环，则对 R 的任意一族 $L-F$ 子集 $\{A_i\}_{i\in S}$，有：

$\models(\forall i)(i\in S\to FR(A_i))\to FR(\bigcap_{i\in S}A_i)$

证明 $[\mathrm{FR}(\bigcap_{i\in S}A_i)]=\prod_{x、y\in R}(\bigcap_{i\in S}A_i(x)\bullet\bigcap_{i\in S}A_i(y)\alpha\bigcap_{i\in S}A_i(-x)\bullet\bigcap_{i\in S}A_i(x+y)\bullet\bigcap_{i\in S}A_i(xy))$
$=\prod_{x、y\in R}(\prod_{i\in S}A_i(x)\bullet\prod_{i\in S}A_i(y)\alpha\prod_{i\in S}A_i(-x)\bullet\prod_{i\in S}A_i(x+y)\bullet\prod_{i\in S}A_i(xy))$
$\geqslant\prod_{x、y\in R}\prod_{i\in S}(A_i(x)\bullet A_i(y)\alpha A_i(-x)\bullet A_i(x+y)\bullet A_i(xy))$
$=\prod_{i\in S}\prod_{x、y\in R}(A_i(x)\bullet A_i(y)\alpha A_i(-x)\bullet A_i(x+y)\bullet A_i(xy))$
$=[(\forall i)(i\in S\to FR(A_i))]$

定理1.4 设 R、T 是两个分明环，$f:R\to T$ 为满同态，则对任意的 $A\in\text{云}_L(R)$，$B\in\text{云}_L(T)$，

1)$\models FR(A)\to FR(f(A))$；2)$\models FR(B)\leftrightarrow FR(f^{-1}(B))$．

证明 1) $[\mathrm{FR}(A)]=\prod_{x、y\in T}(f(A)(x)\bullet f(A)(y)\alpha f(A)(-x)\bullet f(A)(x+y)\bullet f(A)(xy))$
$=\prod_{x、y\in T}(\sum_{f(z)=x}A(z)\bullet\sum_{f(u)=y}A(u)\alpha\sum_{f(v)=-x}A(v)\bullet\sum_{f(w)=x+y}A(w)\bullet\sum_{f(t)=xy}A(t))$
$\geqslant\prod_{x、y\in T}(\sum_{f(z)=x}A(z)\bullet\sum_{f(u)=y}A(u)\alpha\sum_{f(z)=x}A(-z)\bullet\sum_{f(z)=x,f(u)=y}A(z+u)\bullet\sum_{f(z)=x,f(u)=y}A(zu))$
$=\prod_{x、y\in T}(\sum_{f(z)=x,f(u)=y}A(z)\bullet A(u)\alpha\sum_{f(z)=x,f(u)=y}A(-z)\bullet A(z+u)\bullet A(zu))$
$\geqslant\prod_{x、y\in T}(\prod_{z、u\in R}(A(z)\bullet A(u)\alpha A(-z)\bullet A(z+u)\bullet A(zu))$
$=\prod_{z、u\in R}(A(z)\bullet A(u)\alpha A(-z)\bullet A(z+u)\bullet A(zu))=[\mathrm{FR}(A)]$

2)$[\mathrm{FR}(f^{-1}(B))]$
$=\prod_{x、y\in R}(f^{-1}(B)(x)\bullet f^{-1}(B)(y)\alpha f^{-1}(B)(-x)\bullet f^{-1}(B)(x+y)\bullet f^{-1}(B)(xy))$
$=\prod_{x、y\in R}(B(f(x))\bullet B(f(y))\alpha B(f(-x))\bullet B(f(x+y))\bullet B(f(xy)))$
$=\prod_{x、y\in R}(B(f(x))\bullet B(f(y))\alpha B(-f(x))\bullet B(f(x)+f(y))\bullet B(f(x)f(y)))$
$=\prod_{x、y\in T}(B(u)\bullet B(v)\alpha B(-u)\bullet B(u+v)\bullet B(w))=[\mathrm{FR}(B)]$

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L－Fuzzifying Ring

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Abstract: Through semantic method of complete residual lattice－valued logic L, proposed by Professor Mingsheng Ying, the paper introduces the concept of L－fuzzifying ring and discusses some properties of L－ring.

Key words: complete residual lattice－valued logic; L－fuzzifying ring

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